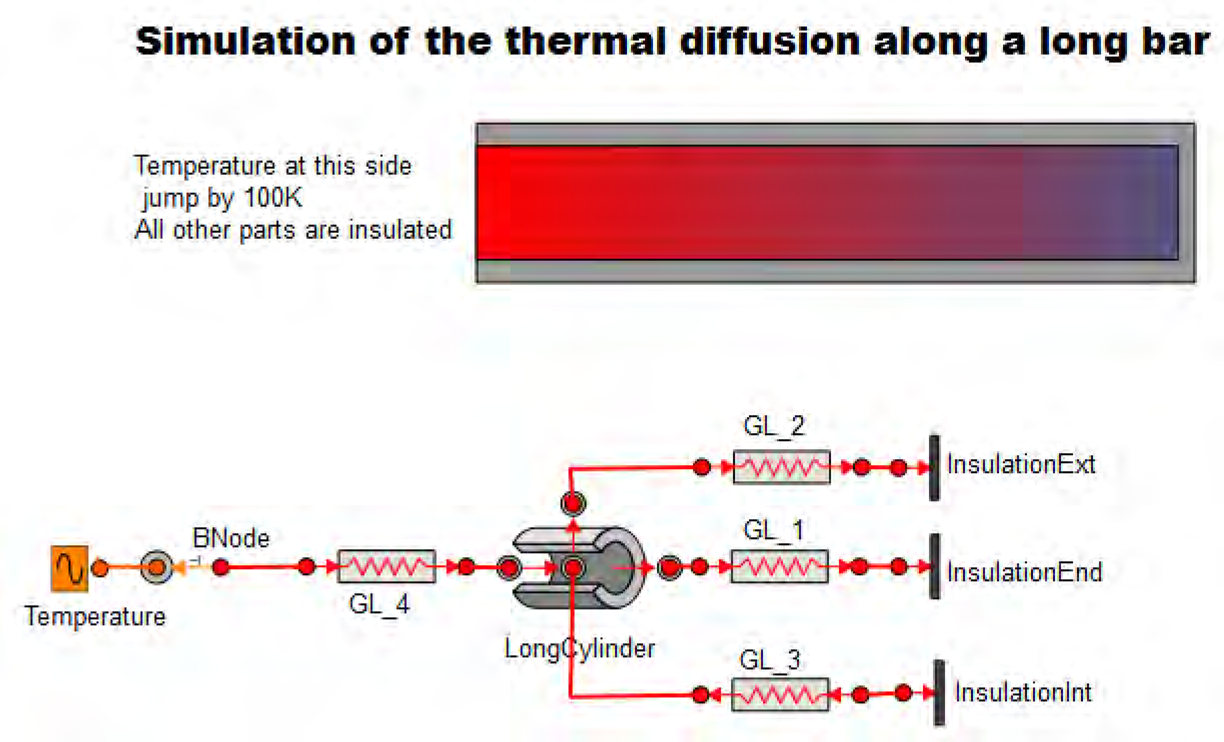
Validation of the diffusion problems : quasi independent of the number of nodes

Reference [R1]: Thermal diffusivity, any book: ENSTA, Transferts thermiques , M. Douchez, ed. 1979.

1 Position of the problem and results

On some simulation, the solution of the diffusion problems is highly dependant on the number of nodes. This short note assesses the effect of the number of nodes for the thermal diffusion for a very simple case of a long insulated bar that has one side suddenly forced to a jump in temperature. Of course at infinite time we expect that the whole bar will reaches the imposed temperature.
The conclusion given by the results of the simulations is that the problem solved by EcosimPro and the thermal library is quasi independent on the number of nodes, including with only 1 node the results are almost correct.

2 Model

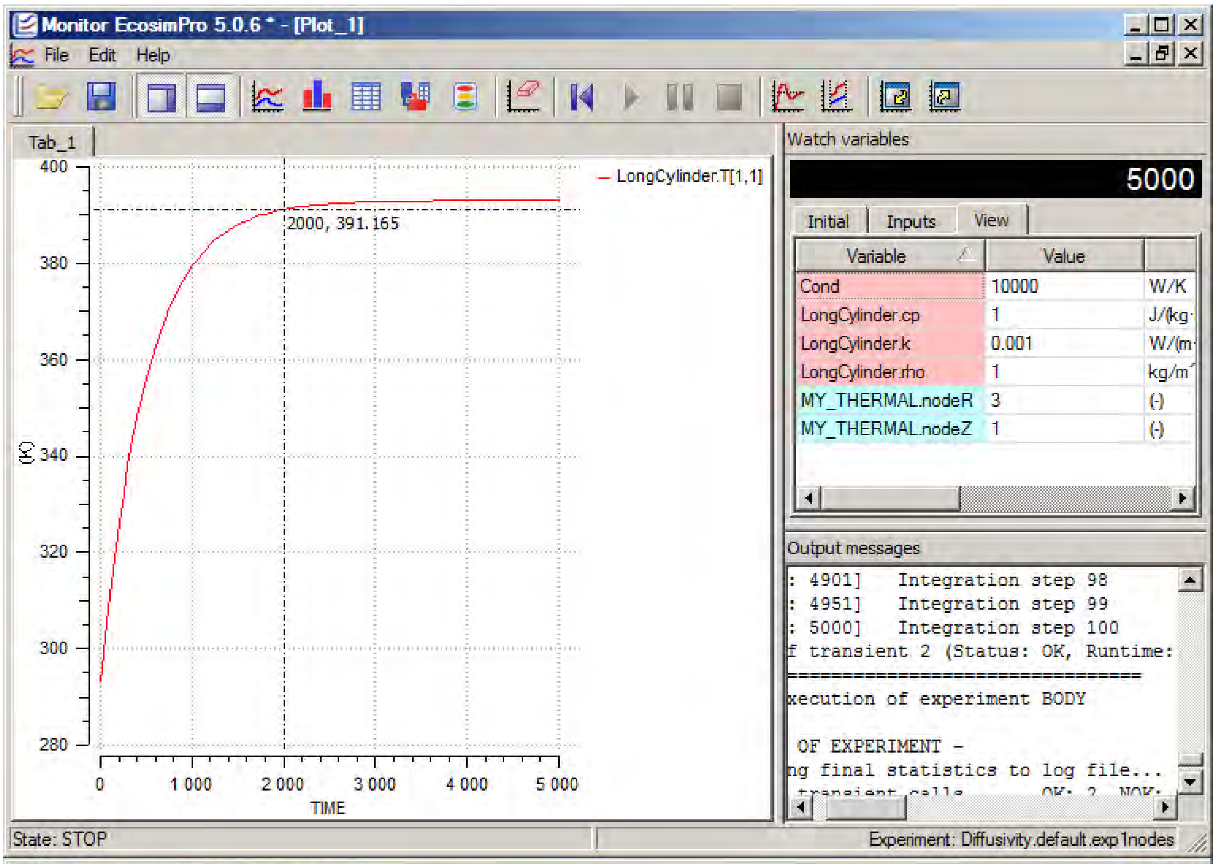


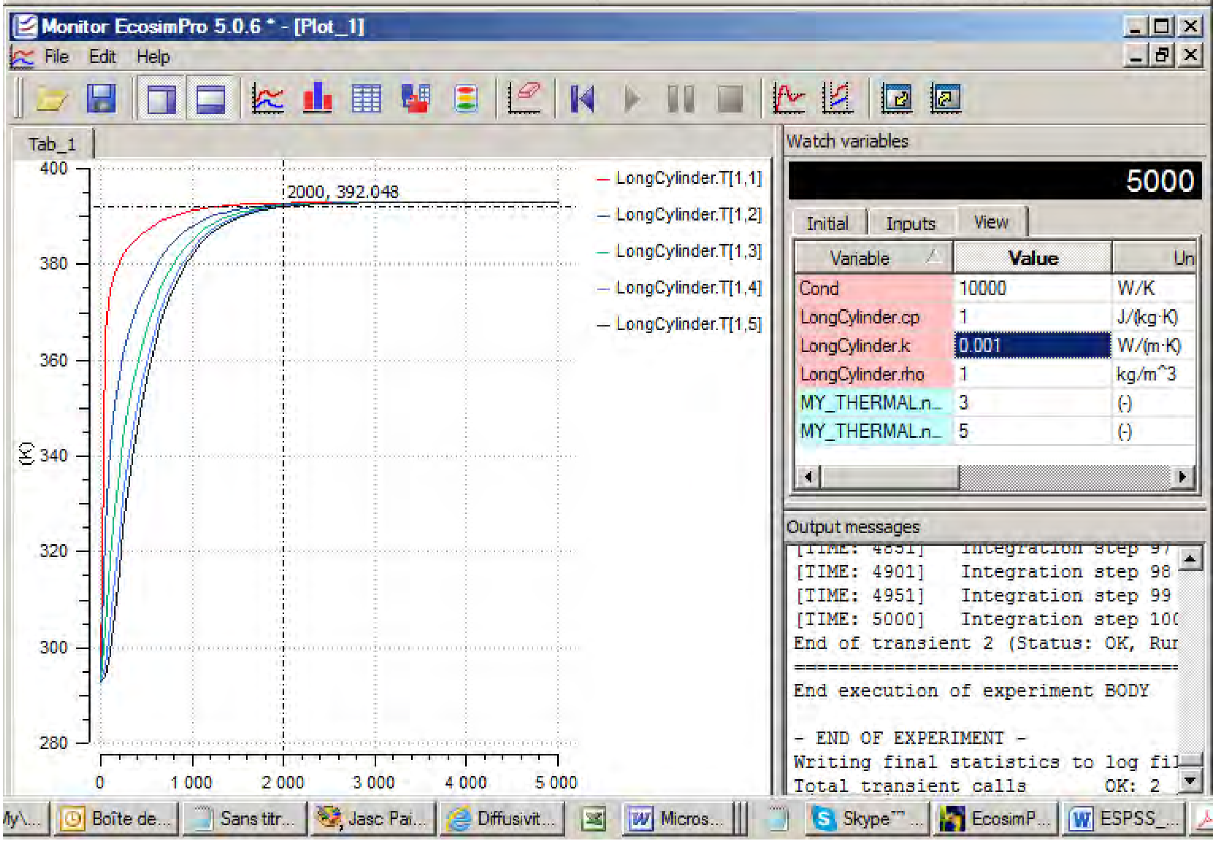
The diffusion coefficient is set to 0.001 (with material of the long cylinder to none with k=0.001, rho=1 and Cv=1).
The length is 1 m.

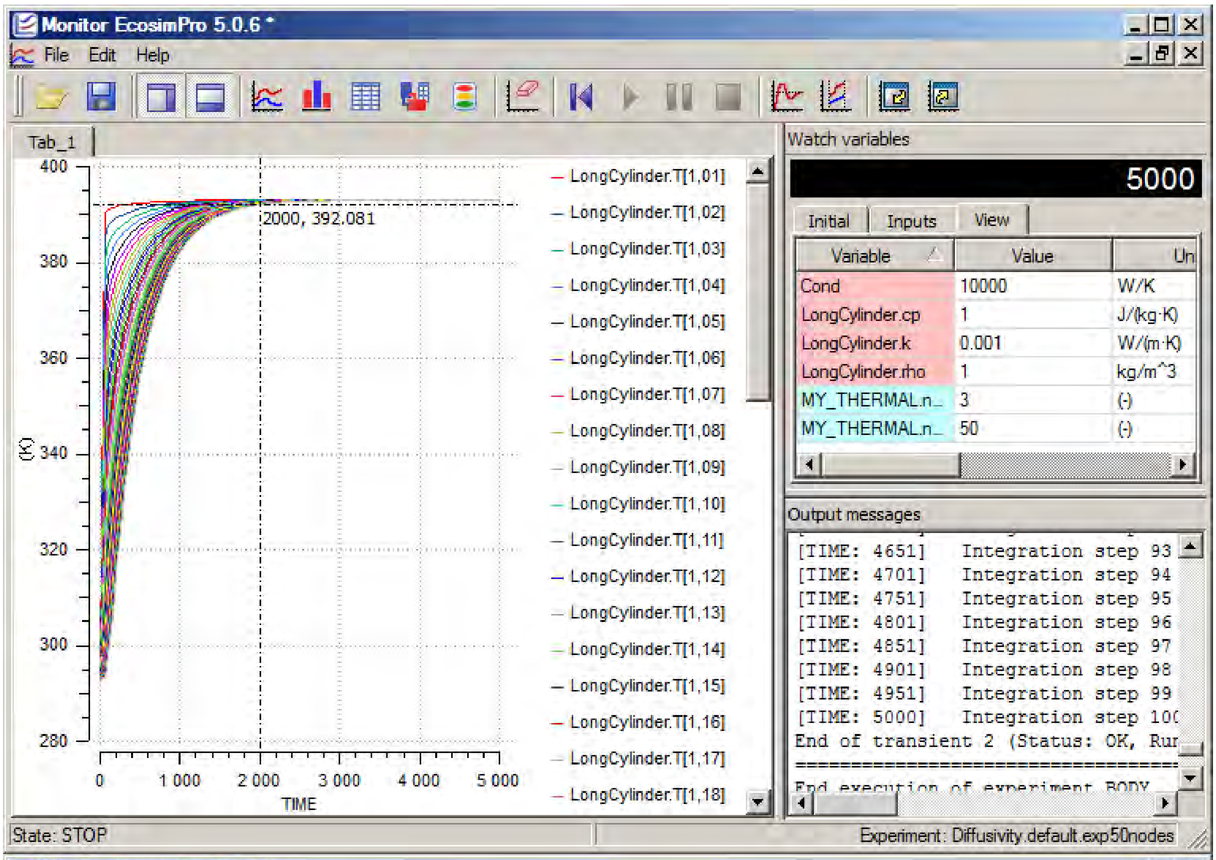
3 Results at the end of the bar (in the last node)

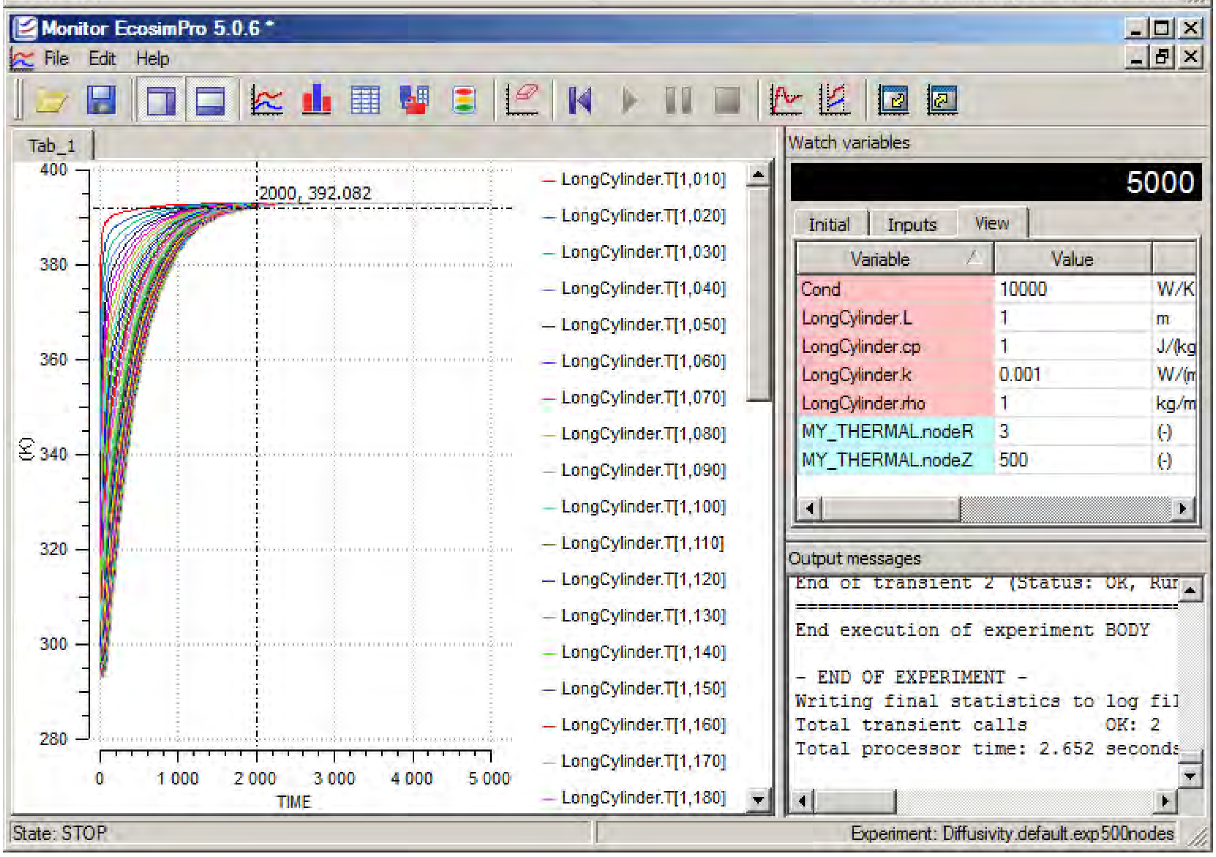
1. At time =2000s, for 1 node: T=391.165 (dT=-1.835 K)
2. At time =2000s, for 5 nodes: T=392.048 (dT=-0.952 K)
3. At time =2000s, for 50 nodes: T=392.081 (dT=-0.919 K)
4. At time =2000s, for 500 nodes: T=392.082 (dT=-0.918 K)

As seen on the plots, the final result at Time =2000s is almost independent on the number of nodes when the nodes vary between 1 and 500.
This is an amazing result because it almost does not depends on the discretisation while the temperature in the node is in the middle of the node and not exactly at the end of the node (this affect mainly the case with 1 node that is slightly overestimated for the end of the bar).

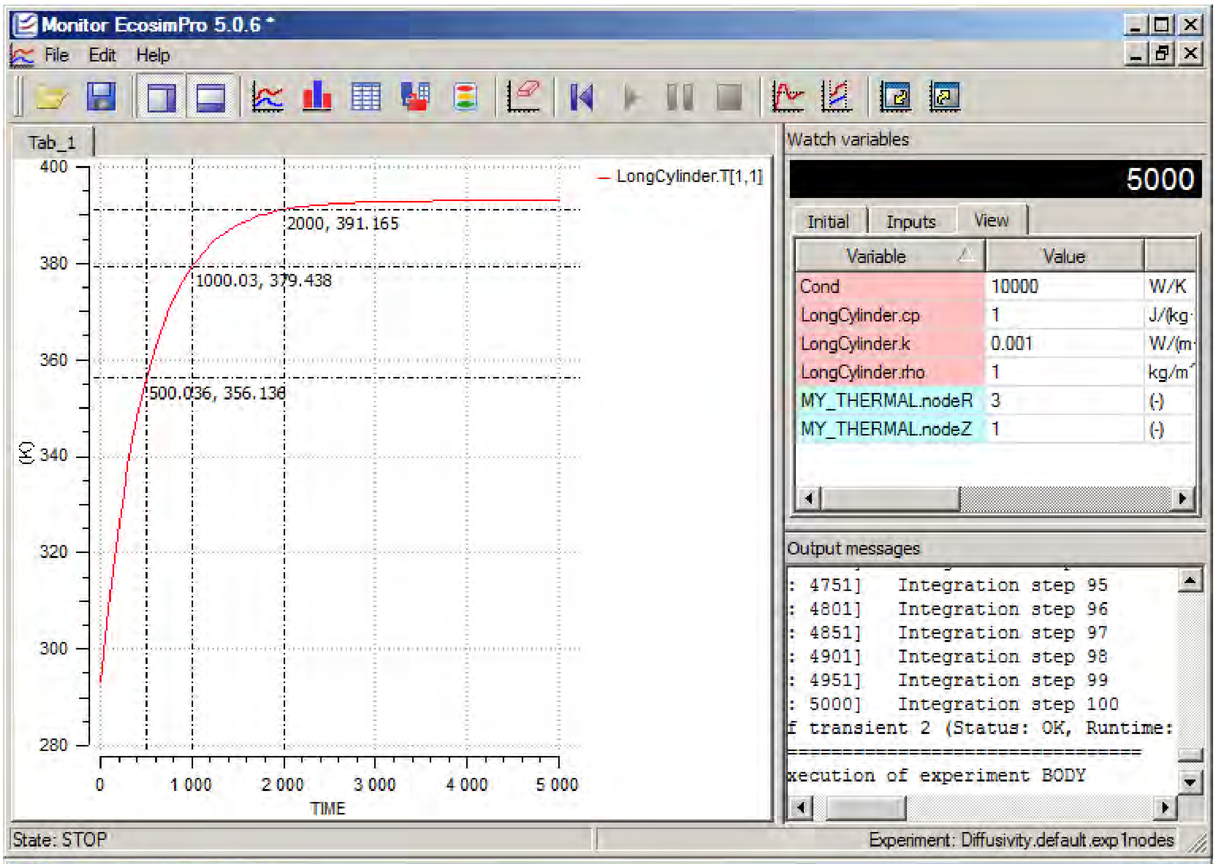

Monitor EcosimPro 5.0.6 * - [Plot_1]
File Edit Help
Tab_1
LongCylinder.T[1,1]
2000, 391.165
(K)
TIME
Watch variables
5000
Initial Inputs View
Variable Value
Cond 10000 W/K
LongCylinder.cp 1 J/(kg·
LongCylinder.k 0.001 W/(m·
LongCylinder.rho 1 kg/m^
MY_THERMAL.nodeR 3 (-)
MY_THERMAL.nodeZ 1 (-)
Output messages
: 4901] Integration step 98
: 4951] Integration step 99
: 5000] Integration step 100
f transient 2 (Status: OK, Runtime:
xecution of experiment BODY
OF EXPERIMENT -
ng final statistics to log file...
State: STOP
Experiment: Diffusivity.default.exp1nodes

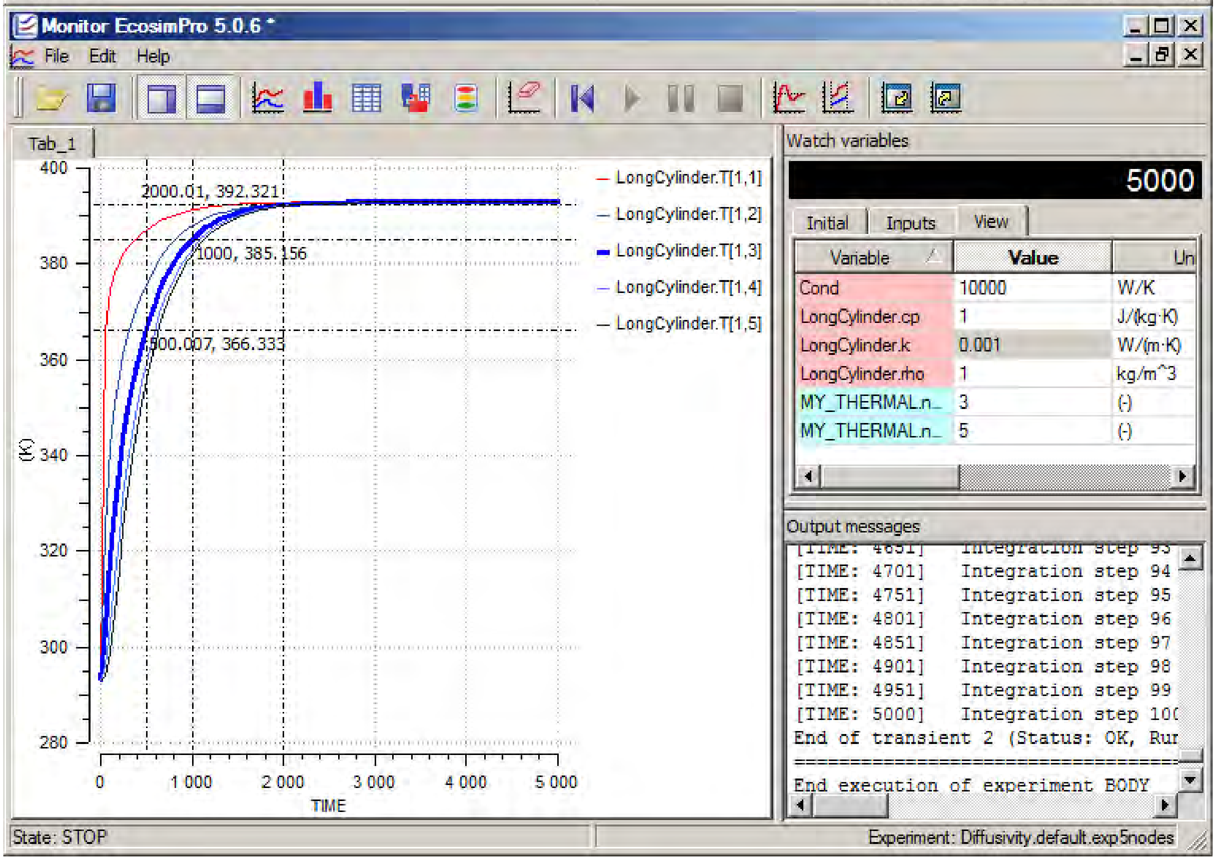

Monitor EcosimPro 5.0.6 * - [Plot_1]
File Edit Help
Tab_1
LongCylinder.T[1,1]
LongCylinder.T[1,2]
LongCylinder.T[1,3]
LongCylinder.T[1,4]
LongCylinder.T[1,5]
2000, 392.048
(K)
Watch variables
5000
Initial Inputs View
Variable Value Un
Cond 10000 W/K
LongCylinder.cp 1 J/(kg·K)
LongCylinder.k 0.001 W/(m·K)
LongCylinder.rho 1 kg/m^3
MY_THERMAL.n... 3 (-)
MY_THERMAL.n... 5 (-)
Output messages
[TIME: 4901] Integration step 98
[TIME: 4951] Integration step 99
[TIME: 5000] Integration step 100
End of transient 2 (Status: OK, Rur
End execution of experiment BODY
- END OF EXPERIMENT -
Writing final statistics to log fil
Total transient calls OK: 2

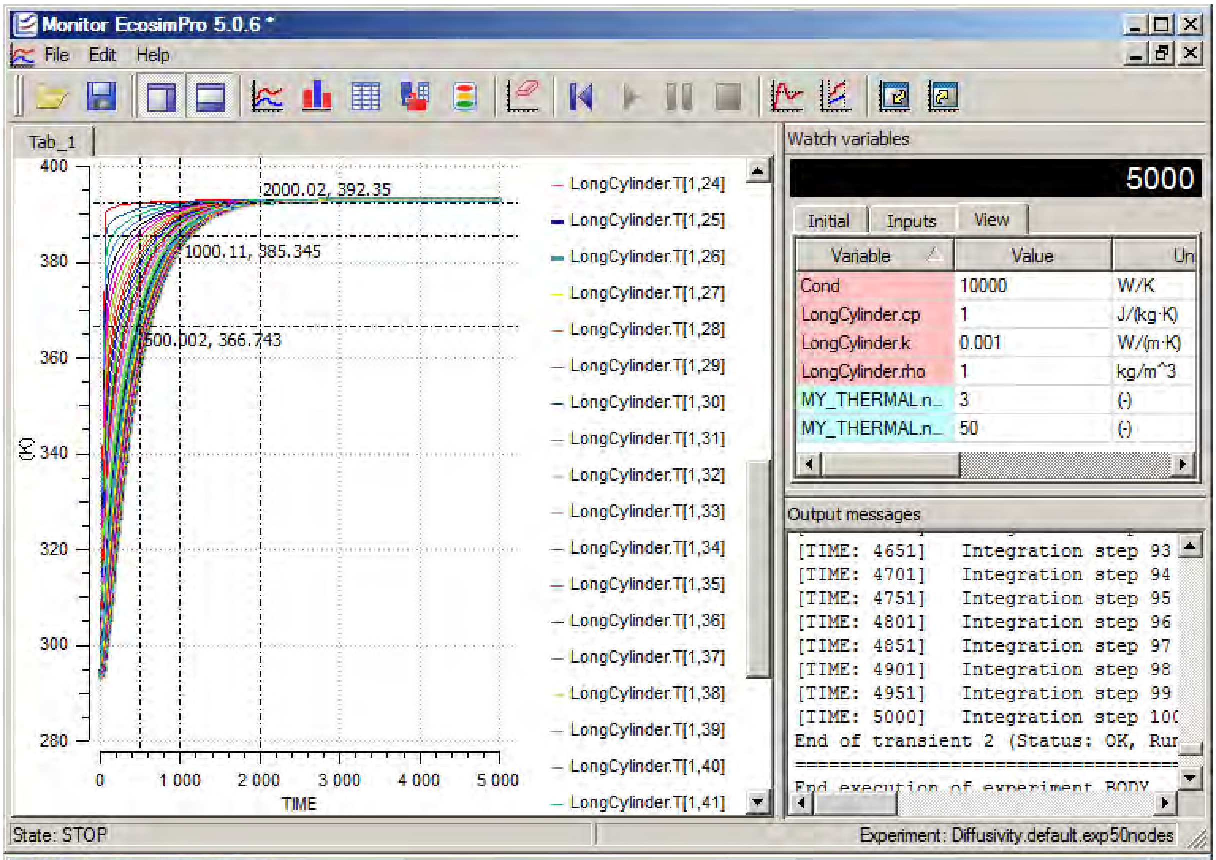

Monitor EcosimPro 5.0.6 *
File Edit Help
Tab_1
400
380
360
(K) 340
320
300
280
2000, 392.081
0 1 000 2 000 3 000 4 000 5 000
TIME
LongCylinder.T[1,01]
LongCylinder.T[1,02]
LongCylinder.T[1,03]
LongCylinder.T[1,04]
LongCylinder.T[1,05]
LongCylinder.T[1,06]
LongCylinder.T[1,07]
LongCylinder.T[1,08]
LongCylinder.T[1,09]
LongCylinder.T[1,10]
LongCylinder.T[1,11]
LongCylinder.T[1,12]
LongCylinder.T[1,13]
LongCylinder.T[1,14]
LongCylinder.T[1,15]
LongCylinder.T[1,16]
LongCylinder.T[1,17]
LongCylinder.T[1,18]
Watch variables
5000
Initial Inputs View
Variable Value Un
Cond 10000 W/K
LongCylinder.cp 1 J/(kg·K)
LongCylinder.k 0.001 W/(m·K)
LongCylinder.rho 1 kg/m^3
MY_THERMAL.n... 3 (-)
MY_THERMAL.n... 50 (-)
Output messages
[TIME: 4651] Integration step 93
[TIME: 4701] Integration step 94
[TIME: 4751] Integration step 95
[TIME: 4801] Integration step 96
[TIME: 4851] Integration step 97
[TIME: 4901] Integration step 98
[TIME: 4951] Integration step 99
[TIME: 5000] Integration step 10(
End of transient 2 (Status: OK, Rur
State: STOP
Experiment: Diffusivity.default.exp50nodes

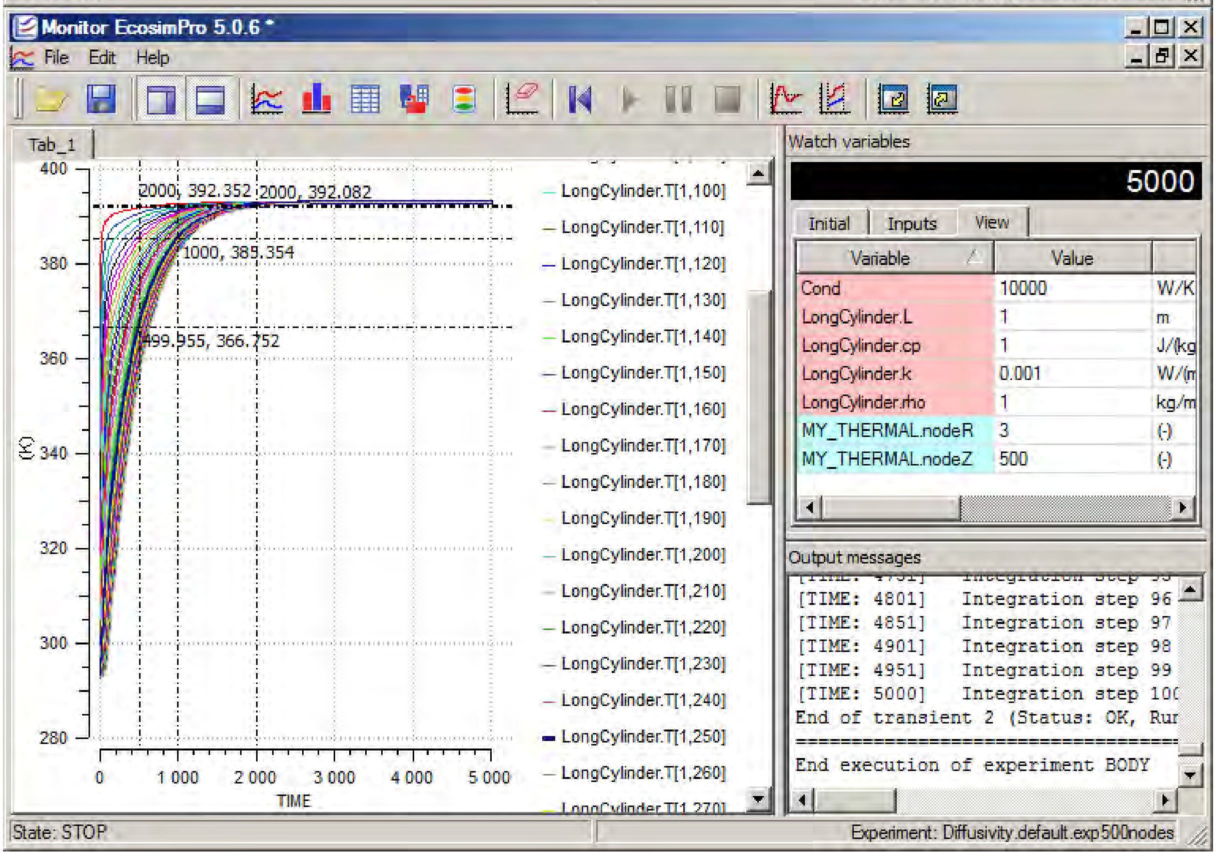

Monitor EcosimPro 5.0.6 *
File Edit Help
Tab_1
400
380
360
(K) 340
320
300
280
2000, 392.082
0 1 000 2 000 3 000 4 000 5 000
TIME
LongCylinder.T[1,010]
LongCylinder.T[1,020]
LongCylinder.T[1,030]
LongCylinder.T[1,040]
LongCylinder.T[1,050]
LongCylinder.T[1,060]
LongCylinder.T[1,070]
LongCylinder.T[1,080]
LongCylinder.T[1,090]
LongCylinder.T[1,100]
LongCylinder.T[1,110]
LongCylinder.T[1,120]
LongCylinder.T[1,130]
LongCylinder.T[1,140]
LongCylinder.T[1,150]
LongCylinder.T[1,160]
LongCylinder.T[1,170]
LongCylinder.T[1,180]
Watch variables
5000
Initial Inputs View
Variable Value
Cond 10000 W/K
LongCylinder.L 1 m
LongCylinder.cp 1 J/(kg
LongCylinder.k 0.001 W/(n
LongCylinder.rho 1 kg/m
MY_THERMAL.nodeR 3 (-)
MY_THERMAL.nodeZ 500 (-)
Output messages
End execution of experiment BODY
- END OF EXPERIMENT -
Writing final statistics to log fil
Total transient calls OK: 2
Total processor time: 2.652 seconds
State: STOP
Experiment: Diffusivity.default.exp500nodes

4 Mid-length results at time 2000s, 1000s and 500s









at time 2000 s

- For 1 node, the middle is located at node 1, thus the temperature at 2000s is 391.165
- For 5 nodes, the middle is located at node 3, thus the temperature at 2000s is 392.32
- For 50 nodes, the middle is located between node 25 and node 26, thus the temperature at 2000s is 392.35

- For 500 nodes, the middle is located between node 250 and node 251, but the difference is so small that the temperature at node 250 is fully valid, thus the temperature at 2000s is 392.352
- Conclusion : except for the case with 1 node, the temperature is not dependant on the number of nodes (<<1 K)

at time 1000s

- For 1 node, the middle is located at node 1, thus the temperature at 1000s is 379.438
- For 5 nodes, the middle is located at node 3, thus the temperature at 1000s is 385.156
- For 50 nodes, the middle is located between node 25 and node 26, thus the temperature at 1000s is 385.345
- For 500 nodes, the middle is located between node 250 and node 251, but the difference is so small that the temperature at node 250 is fully valid, thus the temperature at 1000s is 385.354
- Conclusion : except for the case with 1 node, the temperature is not dependant on the number of nodes (<<1 K)

at time 500s

- For 1 node, the middle is located at node 1, thus the temperature is 356.138
- For 5 nodes, the middle is located at node 3, thus the temperature is 366.33
- For 50 nodes, the middle is located between node 25 and node 26, thus the temperature is 366.743
- For 500 nodes, the middle is located between node 250 and node 251, but the difference is so small that the temperature at node 250 is fully valid, thus the temperature is 366.752
- Conclusion : except for the case with 1 node, the temperature is not dependant on the number of nodes (<1 K)

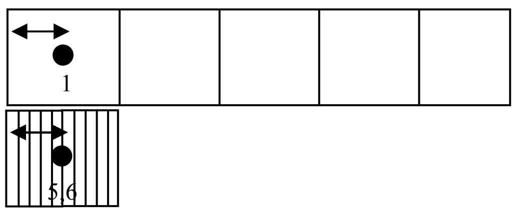
5 Simulation of an infinite bar, and results according to the Erf function

In order to simulate an infinite bar, the total length chosen is 10 m and the analysis is performed at a time for which the last node temperature has almost not changed: that is for time=5000 s and a length 1 m (and the temperature of the last node at 10 m increases by less than 1 K in the case of 5 nodes).

The expected result is given by the complementary error function erfc(x/sqrt(4a.t)) with a=1/1000, t=5000, x=1

$$\mathrm{erfc}\left(\frac{1}{\sqrt{20}}\right) = 0.75183$$

Hence, for a jump of 100 K the expected temperature at x=1 m and Time = 5000 s is 293+75.183= 368.183 K

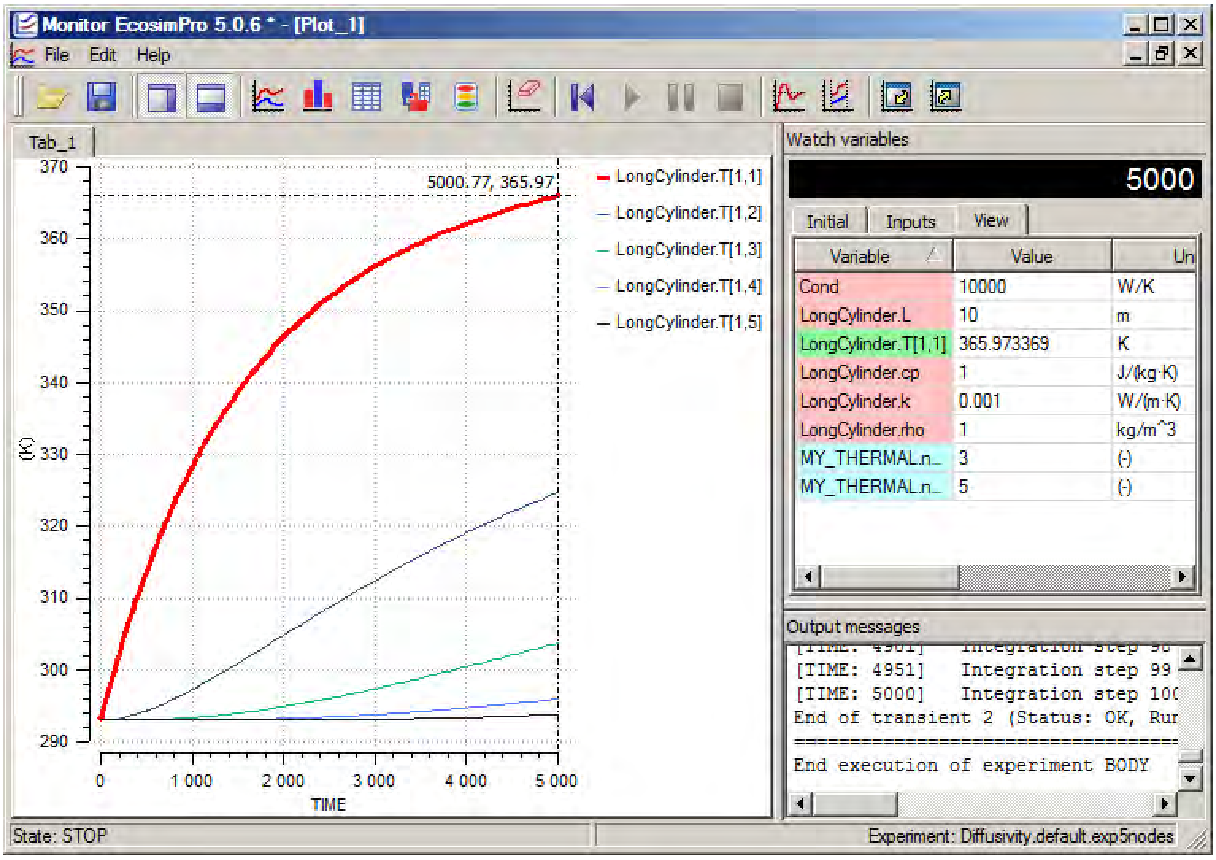


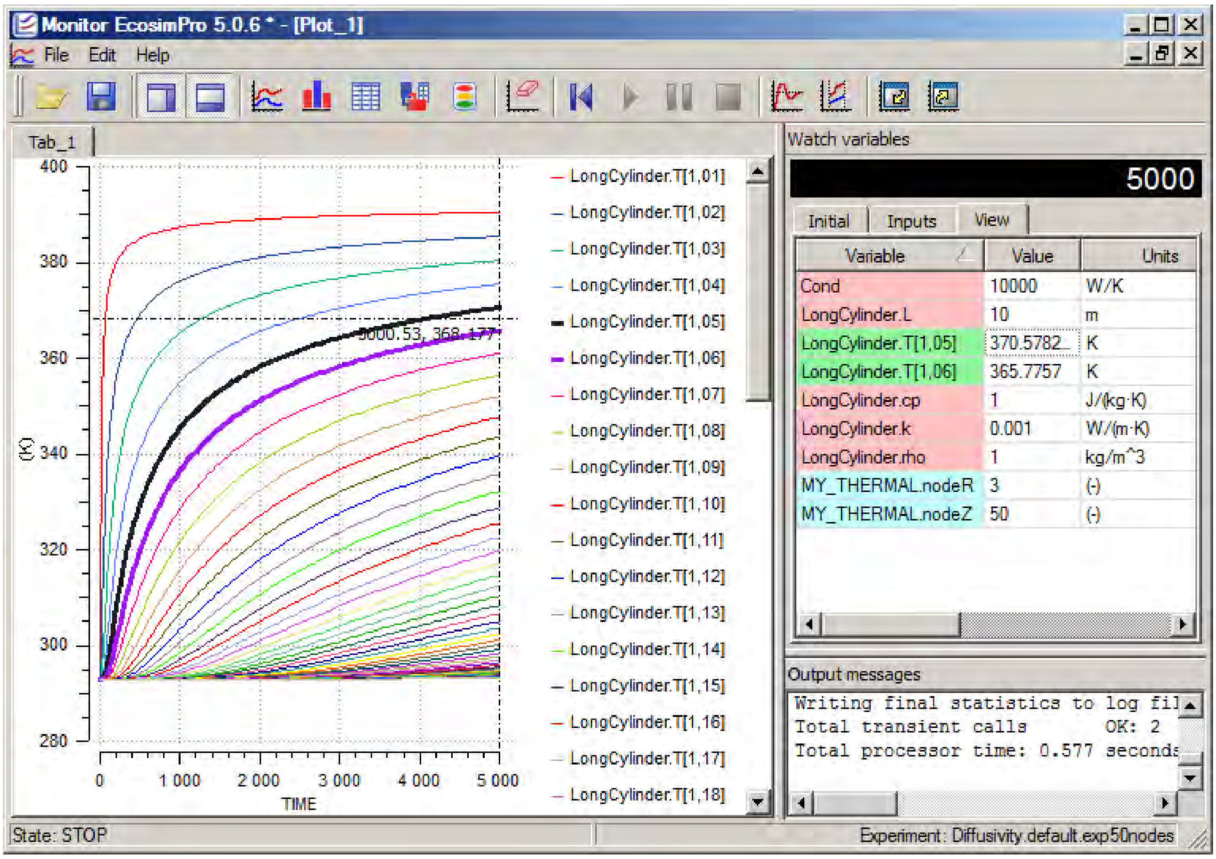
With 5 nodes, the node length is 2 m, but the temperature being given in the middle of each node, for the first node it is at a length 1 m

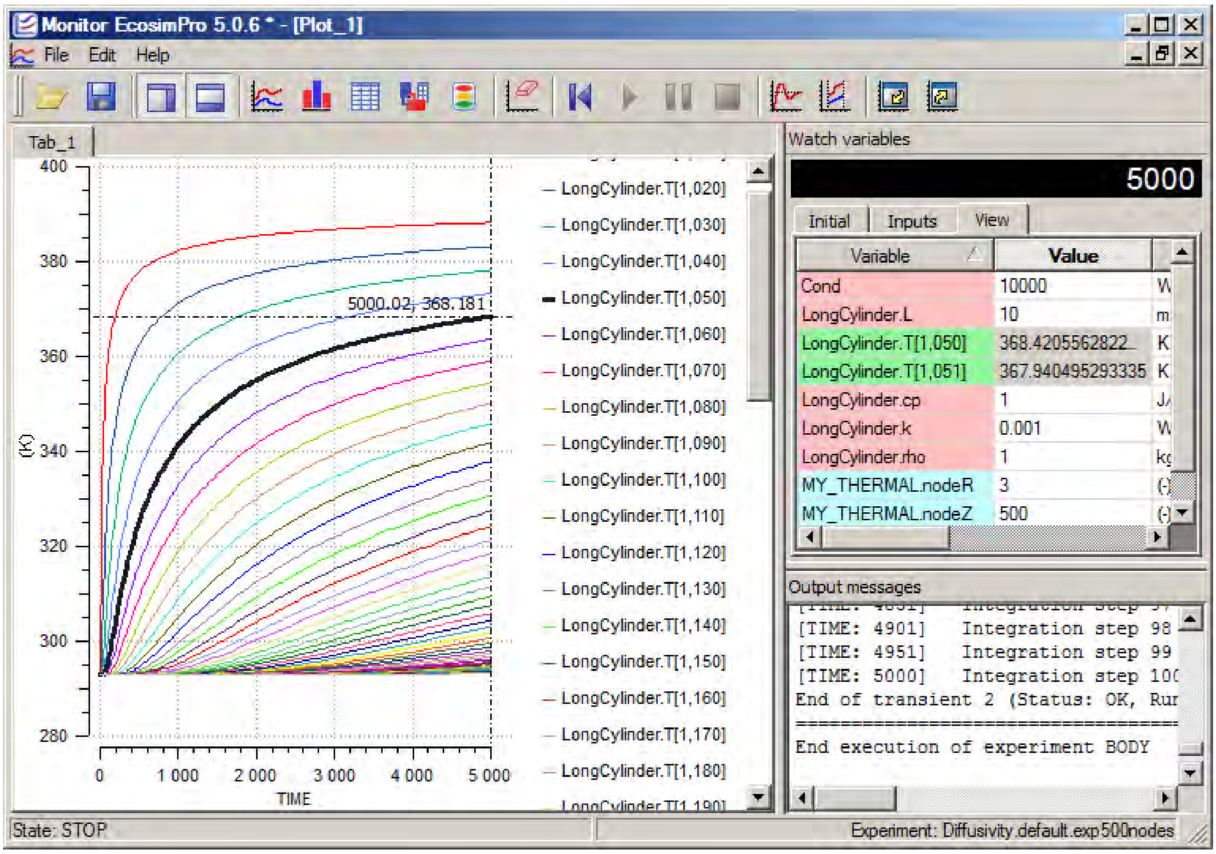
With 50 nodes, the node length is 0.2 m, but for a length of 1 m the temperature is given by the mean nodes temperatures of node 5 and 6

With 5 nodes, T(1m, 5000s)=365.97 (delta T = -2.2 K)
With 50 nodes, T(1m, 5000s)=368.177 (delta T = -0.016 K)
With 500 nodes, T(1m, 5000s)=368.181 (delta T = -0.002 K)

- Conclusion : the validation of the theoretical case of the infinite wall has been performed successfully. Temperature is not very dependant on the number of nodes even for 5 nodes (<2.2 K), but increasing the number of nodes increase the fit.


Monitor EcosimPro 5.0.6 * - [Plot_1]
File Edit Help
Tab_1
5000.77, 365.97
LongCylinder.T[1,1]
LongCylinder.T[1,2]
LongCylinder.T[1,3]
LongCylinder.T[1,4]
LongCylinder.T[1,5]
(K)
TIME
Watch variables
5000
Initial Inputs View
Variable Value Un
Cond 10000 W/K
LongCylinder.L 10 m
LongCylinder.T[1,1] 365.973369 K
LongCylinder.cp 1 J/(kg·K)
LongCylinder.k 0.001 W/(m·K)
LongCylinder.rho 1 kg/m^3
MY_THERMAL.n... 3 (-)
MY_THERMAL.n... 5 (-)
Output messages
[TIME: 4951] Integration step 99
[TIME: 5000] Integration step 100
End of transient 2 (Status: OK, Run
End execution of experiment BODY
State: STOP
Experiment: Diffusivity.default.exp5nodes


Monitor EcosimPro 5.0.6 * - [Plot_1]
File Edit Help
Tab_1
5000.53, 368.177
LongCylinder.T[1,01]
LongCylinder.T[1,02]
LongCylinder.T[1,03]
LongCylinder.T[1,04]
LongCylinder.T[1,05]
LongCylinder.T[1,06]
LongCylinder.T[1,07]
LongCylinder.T[1,08]
LongCylinder.T[1,09]
LongCylinder.T[1,10]
LongCylinder.T[1,11]
LongCylinder.T[1,12]
LongCylinder.T[1,13]
LongCylinder.T[1,14]
LongCylinder.T[1,15]
LongCylinder.T[1,16]
LongCylinder.T[1,17]
LongCylinder.T[1,18]
(K)
TIME
Watch variables
5000
Initial Inputs View
Variable Value Units
Cond 10000 W/K
LongCylinder.L 10 m
LongCylinder.T[1,05] 370.5782... K
LongCylinder.T[1,06] 365.7757 K
LongCylinder.cp 1 J/(kg·K)
LongCylinder.k 0.001 W/(m·K)
LongCylinder.rho 1 kg/m^3
MY_THERMAL.nodeR 3 (-)
MY_THERMAL.nodeZ 50 (-)
Output messages
Writing final statistics to log fil
Total transient calls OK: 2
Total processor time: 0.577 seconds
State: STOP
Experiment: Diffusivity.default.exp50nodes



6 Model traceability

```
-- Generated automatically by - EcosimPro - 5.0.6
USE CONTROL VERSION "3.2"
USE THERMAL VERSION "3.4"
-- ' 15/11/2013 20:46:55
COMPONENT Diffusivity
 DATA
   REAL To=293 UNITS "K"
   REAL Cond=1000 UNITS "W/K"
 TOPOLOGY

   THERMAL.Cylinder(
   nz = nodeZ, -- Non default value.
   nr = nodeR -- Non default value.
   )
    LongCylinder(
   L = 1,
   Di = 0.1,
   Do = 0.2, -- Non default value.
   num = 1,
   mat = None,
   k = 1, -- Non default value.
   rho = 1, -- Non default value.
   cp = 1, -- Non default value.
   TemperatureDependance = TRUE ,
   init_mode = Constant_Temp,
   To = To, -- Non default value.
   To_linear = { 290,290} ,
   To_table = { { 0,1} ,{ 290,290} })

   THERMAL.Insulation(
   InsulationInt except:
   n = nodeR -- Non default value.
   ) InsulationEnd

   THERMAL.Insulation( is_a
InsulationInt)
    InsulationExt

   THERMAL.Insulation(
   n = nodeZ -- Non default value.
   )
    InsulationInt

   THERMAL.BNode(
   n = nodeR -- Non default value.
   )
    BNode(
   Label = "Node Label",
   qi = 0)

   CONTROL.AnalogSource(
   n_out = nodeR -- Non default value.
   )
    Temperature(
   source = Source_Constant,
   Amp = To, -- Non default value.
   Tstart = 0,
   Offset = 0,
   Phase = 0,
   Period = 10,
   pulseWidth = 0.001,
   rampDuration = 10,
   tabmethod = LinearInterpWithEvents,
   timeTable = { { 0,10} ,{ 0,0} })

   THERMAL.GL( is_a GL_4)
    GL_1(
    is_a GL_4)

   THERMAL.GL(
   GL_4 except:
   n = nodeZ -- Non default value.
   ) GL_2(
    is_a GL_4)

   THERMAL.GL(
   GL_4 except:
   n = nodeZ -- Non default value.
   ) GL_3(
    is_a GL_4)

   THERMAL.GL(
   n = nodeR -- Non default value.
   )
    GL_4(
   cond = Cond -- Non default value.
   )
   CONNECT Temperature.s_out TO
 BNode.s_temperature
   CONNECT LongCylinder.tpz_out TO
 GL_1.tp_in
   CONNECT GL_1.tp_out TO
 InsulationEnd.tp
   CONNECT GL_2.tp_out TO
 InsulationExt.tp
   CONNECT LongCylinder.tpr_out TO
 GL_2.tp_in
   CONNECT InsulationInt.tp TO
 GL_3.tp_in
   CONNECT GL_3.tp_out TO
 LongCylinder.tpr_in
   CONNECT GL_4.tp_out TO
 LongCylinder.tpz_in
   CONNECT GL_4.tp_in TO
 BNode.tp_in
END COMPONENT
```

```
-- '  15/11/2013  20:48:25
EXPERIMENT exp500nodes ON Diffusivity.default
 DECLS
   OBJECTS
 INIT
   -- initial values for state variables
   BOUNDS
 BODY
   -- report results in file reportAll.rpt
   --REPORT_TABLE("reportAll.rpt", "*")
   -- integrate the model 15 seconds and obtain results every 0.1 seconds
   RDIGITS=16
   ABS_ERROR=1E-4
   REL_ERROR=ABS_ERROR
   REPORT_MODE=IS_STEP -- REPORT_MODE=IS_EVENT,IS_CINT,IS_STEP
   TIME = 0
   INTEG_TO(1,0)
   LongCylinder.k=0.001
   LongCylinder.L=1
   To=393
   Cond=10000
   INTEG_TO(5/LongCylinder.k,5/LongCylinder.k/100)
   --INTEG_TO(1000,10)
   --INTEG_TO(10000,100)
   --INTEG_TO(100000,1000)
END EXPERIMENT
```