Short note on the performances of Magnetic torquers (Magneto-torquers) MTQ

It will be shown that a surprising simple relation can be set for MTQs:
That is for Cubesat size in LEO, the cost of MTQs is surprisingly 14 µNm/kg
How come?

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Ref. 1 Florian Reichel, Philip Bangert, "DESIGN, TEST AND VERIFICATION OF A MINIATURE ATTITUDE CONTROL SYSTEM FOR THE PICOSATELLITE UWE-3," IAC-13,E2,2,6,x19006
Ref. 2 Jozef Van Der Ha (personal communications)

1 Why MTQ's?

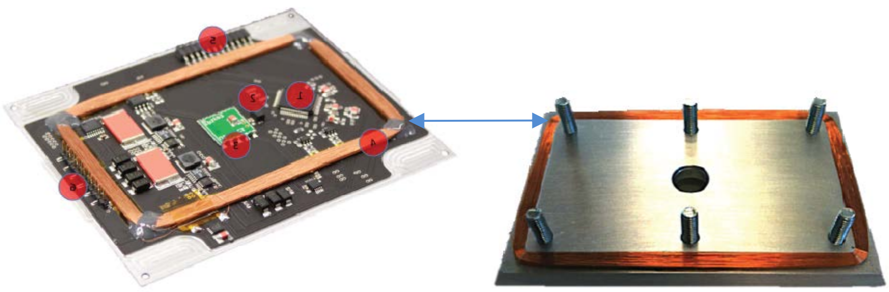
When considering satellites, MTQ's are employed for the purpose of attitude control with the help of the Earth's magnetic field. The generated torque must be able to cancel disturbing torques (e.g., those induced by solar radiation pressure or air drag) acting on the satellite.
Usually, MTQ's have 3 independent magnetic moments (along 3 directions in the satellite body) and the generated torque vector $\vec{T}$ is determined by the available magnetic field strength $\vec{B}$ along the satellite orbit.
The essential design parameters are the magnitudes of the 3 independent components of the magnetic moment vector $\vec{M}$. Inside a magnetic field, the MTQ produces a torque according to:

$$\vec{T} = \vec{M} \times \vec{B} \qquad [1]$$

This equation [1] is the most essential physical relationship for studying attitude control by MTQ's. The $\vec{B}$ vector is determined by the Earth's magnetic field along the satellite's orbit and cannot be changed. The vector $\vec{M}$ can be designed in the following way
a) Before launch by changing the design of the coil lengths, thickness, and coil areas
b) In-orbit by varying the currents flowing through the 3 coils.

2 What is a MTQ?

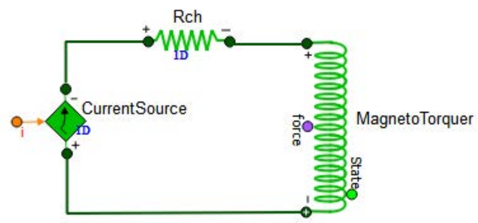
Here in this short note, one consider flat coils like in the pictures below (courtesy of ref. 1)



3 How MTQ works?

The MTQ are a so simple device that is it hard to find relevant information, rules, design parameters for having an idea of their performances and cost in terms of mass.

The flat coil is fed by a source current *I* and the MTQ produce a magnetic moment $\vec{M} = I.\overrightarrow{dS}$ where $\overrightarrow{dS}$ is the area loop, a vector oriented normal to the current loop.



For a coil, the amplitude of $\overrightarrow{dS}$ is $N_{turn}.A_{coil}$ where A_{coil} is the area of the coil, so one has with $\overrightarrow{e_n}$ the unit normal vector of the coil:

$$\vec{M} = I.N_{turn}.A_{coil}.\overrightarrow{e_n} \quad [2]$$

We may write: $\vec{M} = \vec{M}(I, N_{turn}, A_{coil})$ this is a 3 dimensions space. Constraining the product "$I.N_{turn}.A_{coil}$" to be a constant fixes of course the value of the magnetic moment $\vec{M}$, hence for a given $\vec{M}$ there are still <u>two degrees of freedom</u> for designing a MTQ.

4 How to design a MTQ at best

The equation [2] in module give $M = I.N_{turn}.A_{coil}$, but this does not say anything about how to optimise (in the sense of having the lowest mass, the lowest current *I* --and more pragmatically the lowest power *P* for a low consumption and a low heating-- for a given *M)*.

So one shall detail those parameters:

- First, the mass:

$$Mass = \pi r^2.N_{turn}.\lambda.\rho_d \quad [3]$$

  where λ is the length of a single loop (the perimeter of the coil so that the total length of the copper is $N_{turn}.\lambda$); ρ_d = density of copper and r radius of the copper wire.

- The power (with U the voltage across the MTQ):

$$Power = U.I \qquad \text{i.e} \qquad Power = {U^2}/{R} \quad [4]$$

- The resistance (where ρ_r = electric resistivity of copper)

$$R = {\rho_r.N_{turn}.\lambda}/{\pi r^2}. \quad [5]$$

First, one found with $I = {U}/{R} = {U\pi r^2}/{\rho_r.N_{turn}.\lambda}$ that:

$$M = I.N_{turn}.A_{coil} \qquad \rightarrow \qquad \boxed{M = \frac{U\pi r^2}{\rho_r}.\frac{A_{coil}}{\lambda}} \quad [6]$$

So again, constraining the product "$Ur^2 A_{coil}/\lambda$" to be a constant fixes the value of the magnetic moment $\vec{M}$.

But also, the magnetic moment $\boldsymbol{M}$ can be written using directly the *Power* and *Mass* product because

$$\mathrm{Power.Mass} = \left(\mathrm{U}.\frac{\mathrm{M}}{\mathrm{N_{turn}.A_{coil}}}\right).(\pi r^2.\mathrm{N_{turn}}.\lambda.\rho_d)$$

$Power.Mass = \frac{U.\pi r^2.\lambda.\rho_d.M}{A_{coil}}$ and with $M = \frac{U\pi r^2 A_{coil}}{\lambda.\rho_r}$ one replaces $U\pi r^2 \rightarrow Power.Mass = \frac{M^2.\lambda^2.\rho_d.\rho_r}{A_{coil}{}^2}$

So one get a very remarkable physical result:

$$\boxed{\mathrm{M} = \frac{1}{\sqrt{\rho_d.\rho_r}}.\frac{\mathrm{A_{coil}}}{\lambda}.\sqrt{\mathrm{Power.Mass}}} \quad [7]$$

Once more, constraining the product "$\sqrt{Power.Mass}.(A_{coil}/\lambda)$" to be a constant fixes the value of the magnetic moment $\vec{M}$.

Note: Once for a given flat coil dimension "A_{coil}, λ" its power $\boldsymbol{P}$ *can be computed, the current density* $\boldsymbol{J}$ *in the wire is fixed:*

$$J = \frac{I}{\pi r^2} = \frac{Power}{U\pi r^2} = \frac{Power.A_{coil}}{\lambda.\rho_r.M} \ \mathrm{A/m^2} \quad [8]$$

5 Relation between Power and Mass for a MTQ

Actually the power has a cost in terms of mass: one cites for example that for Cubesats, 148g of solar arrays can provide at BOL 2W (https://www.clyde.space/products/33-2u-singledeployable-solar-array-long-edge): so the specific power (mass-to-power ratio also called "alpha" ratio in kg/kW) is 0.148/2 kg/W i.e. alpha= 74 kg/kW, either one have other figures of alpha= 50kg/kW at BOL(https://www.clyde.space/products/27-2u-doubledeployed-solar-array).

With a reasonable figure for the entire life of alpha = 64 kg/kW (the designer can choose to include more mass per kW for dealing with other impacts of power like thermal constrains higher when the power get higher, marginal mass of the dedicated power converter), one can simply write

$$\sqrt{Power.Mass} = \sqrt{\frac{1}{64/1000} Mass_{Power}.Mass} = \frac{\lambda.\sqrt{\rho_d.\rho_r}}{A_{coil}}.M$$

$$\sqrt{Mass_{Power}.Mass} = \frac{\lambda.\sqrt{0.064.\rho_d.\rho_r}}{A_{coil}}.M$$

For example for the case of Cubesats, with a coil dimension of 52 x 85 mm, A_{coil} = 0.00442 m²
λ = 0.274 m and with the data $\rho_d = 8920 \ \mathrm{kg/m^3}$, $\rho_r = 1.8\mathrm{E}\text{-}8\ \Omega.\mathrm{m}$ one get:

$$\sqrt{Mass_{Power}.Mass} = 0.2\ M \quad [9]$$

Note : For other shapes of the MTQ, it is obvious to see that a square of side D or a circle with same diameter D produce both the same ratio $A_{coil}/\lambda = D/4$ and thus for such cases with D =64 mm on has the same result: $\sqrt{Mass_{Power}.Mass} = 0.2\ M$.

6 Optimisation wrt the total mass

Now, with eq. [9] it is possible to optimise wrt the total mass. The minimum total mass (mass of the MTQ coil itself plus the mass of the dedicated power part needed to feed the MTQ) is obtained when both masses are equal (from basic know-how: the sum x+y constrained to a fixed product x.y is minimum for y=x). Hence one has:

$$\frac{1}{2}TotalMass = 0.2\ M \qquad \frac{\boldsymbol{M}}{\boldsymbol{TotalMass}} = \mathbf{2.5}\ \mathrm{A.m^2/kg}$$

For 3 axis control, that is at least 3 MTQ's, so: $\frac{M}{SystemMass} = \frac{2.5}{3}$ A.m²/kg [10]

Note : a)This result is also independent on the shape of the coil (either circular or square of same dimension D, the value 2.5 refer to D = 64 mm and is proportional to D)
b)Adding the relationship between Power and mass thanks to the parameter alpha (kg/kW) and stating that this mass shall be equal to the MTQ coil mass (optimum minimum total mass) uses one of the two degrees of freedom of the system design.
c) But still it remains one degree of freedom: either N_{turn} or U or r can be freely chosen to design the MTQ at the best of manufacturing constrains (a given value of r or N_{turn}) or at the best of operational constrains (a given value of U)... At this stage one can say that surprisingly, the magnetic moment M can be seen as independent on the number of turn N_{turn} because N_{turn} can be freely chosen --*but of course the other design parameters must changes accordingly*--.

However, with such optimisation the $Mass_{Power}$ is known from the constrains added, so the power is also known by parameter alpha (kg/kW), hence the current density in the copper wire is fixed by eq.[8]

$$J = \frac{I}{\pi r^2} = \frac{Power.A_{coil}}{\lambda.\rho_r.M} = \frac{\frac{1}{2}TotalMass.A_{coil}}{0.064\ \lambda.\rho_r.TotalMass.2.5} = \frac{\frac{1}{2}.A_{coil}}{0.064\ \lambda.\rho_r.2.5} = \ 2.8\text{E6 A/m}^2 \rightarrow J = 2.8 \text{ A/mm}^2.$$

Constraining J to a different value (lower or even higher) will reduce the value of the factor 2.5 in [10] and make the MTQ's less efficient in term of mass. *But one can select an aluminium wire to get lower current density and lower overall mass too.*

7 Earth magnetic field B

The Earth magnetic field along a circular orbit inclined at 50° at 500 km has components of effective strength of $40\mu N/\sqrt{2}$ down toward Earth (axis Z); $20\mu N/\sqrt{2}$ along the velocity (axis X) and about $10\mu N$ along the transverse axis Y. Division by $\sqrt{2}$ account for quasi sinusoidal shape of the field according to the plot Figure 1.

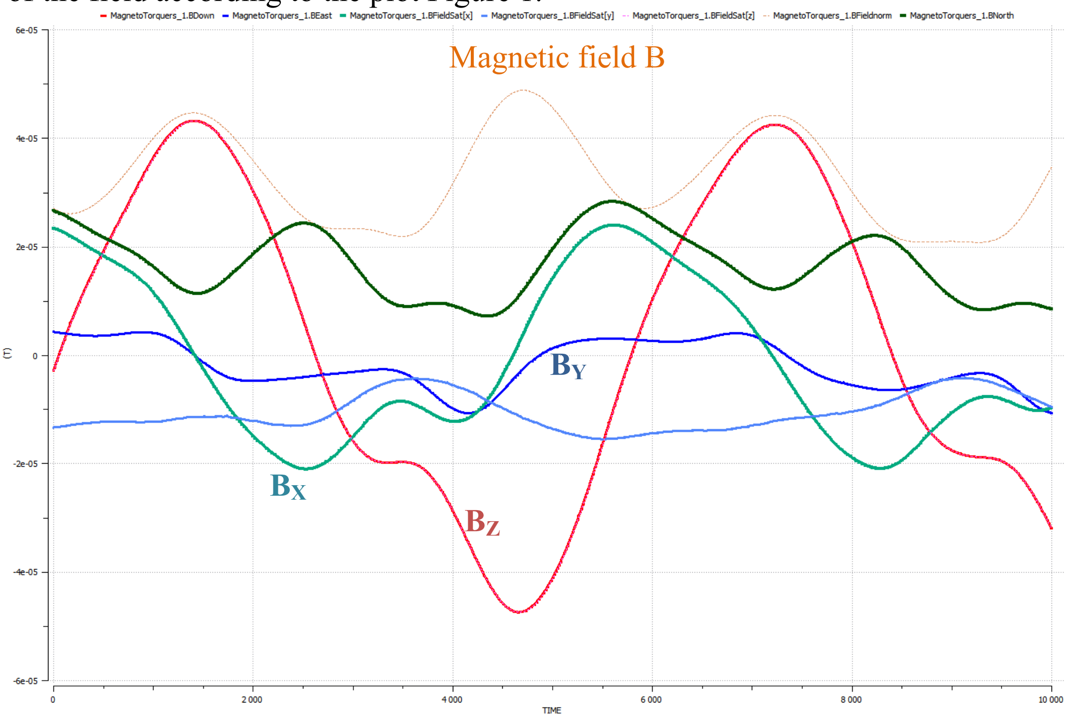


Figure 1: Earth B in East, North, Down frame and in satellite body frame (X, Y, Z) along circular orbit 50°, 500 km

Hence roughly, one can say that the torque produced by a MTQ of magnetic moment M with an axis perpendicular to Z is about M.40/1.414 µNm **T≈28M** µNm, but for other cases one have **T≈14M** µNm or **T≈10M** µNm: that gives a mean of

$$\boldsymbol{T} \approx \mathbf{17}\boldsymbol{M}\ \mu\text{Nm} \qquad [11]$$

8 Surprising conclusion:

For a single MTQ, one has seen from eq. [11]: $T \approx 17M$ µNm, and in combination with eq. [10] for the 3 axis system, $\frac{M}{SystemMass} = \frac{2.5}{3}$ A.m²/kg on get:

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$$\boxed{\frac{\boldsymbol{T}}{\boldsymbol{SystemMass}} \approx \mathbf{14}\ \mu\text{Nm/kg}} \qquad [12]$$

Note: This is valid for the case of the Cubesat size (according to $M = \frac{U\pi r^2 A_{coil}}{\lambda.\rho_r}$ which depends on the coil size) optimized in terms of total mass with an "alpha" of 64 kg/kW, and for a mean magnetic field in LEO around 500 km circular orbit inclined at 50° with a maximum current density in the copper wire of J=2.8 A/mm².

And as seen before, there is still one degree of freedom for designing such MTQs (the radius of the wire for example can be fixed without changing anything in the so much surprising eq. [12]).
