Memo on the Radiation matrix from radiosity for grey emitting surfaces, C. R. Koppel, KopooS Consulting ind. (c) 2000-2013 www.KopooS.com

## Main hypothesis

Uniform Radiosity

1) Uniform temperature surface of each element (node)
2)Grey surfaces, grey body : $\varepsilon_i = \alpha_i$ that is $\rho_i = 1 - \varepsilon_i = 1 - \alpha_i$
3)Diffusely radiated Reflexion
4)Diffusely radiated Emission
5)No Transmission

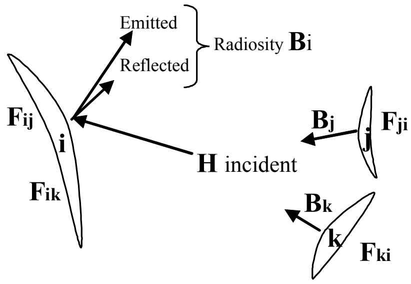
**Nota**:
Reflected $\rho$
Absorbed $\alpha$
Emitted $\varepsilon$
Transmitted $\tau$
and $\alpha+\rho+\tau=1$

Non intuitive notations $F_{ij}$ the angular view factor from ***i*** toward ***j*** (see properties), $A_i$ the surface (m²)

**Net rate of heating** (W) for the element ***i*** (rule Qi: positive when energy is leaving element **i** )

$$Q_i = \text{Emitted} - (\text{Incident} - \text{Reflected})(\text{W})$$

$$Q_i = \text{Emitted} - \text{Absorbed from Incident Radiation}(\text{W}) \qquad \rho_i = 1 - \varepsilon_i = 1 - \alpha_i$$




***1)*** $$Q_i = A_i\cdot\varepsilon_i\cdot\sigma\cdot(T_i)^4 - A_i\cdot\alpha_i\cdot H_i$$

$H_i$: **Incident Radiant Heat flux** on the element ***i*** due to the other elements ***j*** (W/m²)
(rule Hi: positive when energy is arriving on **i** , thus positive as when leaving other **j** )

**For pure black bodies** $$H_i = \sum_j\left[\varepsilon_j\cdot\sigma\cdot(T_j)^4\cdot A_j\cdot F_{ji}\cdot\frac{1}{A_i}\right]$$ With reciprocity rule, $$H_i = \sum_j\left[\varepsilon_j\cdot\sigma\cdot(T_j)^4\cdot F_{ij}\right]$$

$$Q_i = A_i\cdot\sigma\cdot(T_i)^4 - A_i\cdot H_i$$

Black body emissive power =Heat power Emitted (W/m²) = $\sigma\cdot(T_j)^4$ with $\varepsilon_j = 1$. Because of $\sum_j F_{ij} = 1$, one have $(T_i)^4 = \sum_j\left[(T_i)^4\cdot F_{ij}\right]$

Thus : ***1bis)*** $$Q_i = \sum_j\left[A_i\cdot F_{ij}\cdot\sigma\cdot\left[(T_i)^4 - (T_j)^4\right]\right]$$

**For gray bodies**, by analogy, the black body emissive power into $\mathbf{H}_i$ is replaced by the **Radiosity** $B_j$(W/m²) =Heat power **Emitted +Reflected**
(rule Bi: idem rule Hi )

***2)*** $$H_i = \sum_j(B_j\cdot F_{ij})$$ with $$B_j = \varepsilon_j\cdot\sigma\cdot(T_j)^4 + \rho_j\cdot H_j$$ Note: this definition is implicit because $B_j$ is also a function of $B_k$ and itself $B_j$.

***3)*** That gives also for the element *i* $$B_i = \varepsilon_i\cdot\sigma\cdot(T_i)^4 + \rho_i\cdot H_i$$ hence $$B_i = \varepsilon_i\cdot\sigma\cdot(T_i)^4 + (1-\varepsilon_i)\cdot\left[\sum_j(B_j\cdot F_{ij})\right]$$ and $$H_i = \frac{B_i - \varepsilon_i\cdot\sigma\cdot(T_i)^4}{(1-\varepsilon_i)}$$

***4)*** $$\sum_j(X_{ij}\cdot B_j) = \sigma\cdot(T_i)^4$$ with $$X_{ij} = \frac{\delta_{ij} - (1-\varepsilon_i)\cdot F_{ij}}{\varepsilon_i}$$ Note: Kronecker symbol : $\delta_{ii} = 1$; $\delta_{ij} = 0$ for $i \neq j$

With matrix notation, $(B)$, $(\sigma T^4)$ being columns matrixes;
from **4**) view factors & emissivity, matrix $(X)$, its inverse $(X^{-1})$

$$(X)\cdot(B) = (\sigma T^4) \text{ thus } (B) = (X^{-1})\cdot(\sigma T^4) \text{ i.e. } B_i = \sum_j\left[(X^{-1})_{ij}\cdot\sigma\cdot(T_j)^4\right]$$

Bi is known when Tj are known

**Equations *1)* and *3) for Hi* give** $$Q_i = \frac{A_i\cdot\varepsilon_i}{1-\varepsilon_i}\cdot\left[\sigma\cdot(T_i)^4 - B_i\right]$$

***5)*** That gives for open configurations $$Q_i = \sum_j -\left[A_i\cdot\Lambda_{ij}\cdot\sigma\cdot(T_j)^4\right]$$ with $$\Lambda_{ij} = \frac{\varepsilon_i}{1-\varepsilon_i}\cdot\left[\delta_{ij} - (X^{-1})_{ij}\right]$$

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***6)*** **Finally,** because $\sum_j \Lambda_{ij} = 0$, **for all *i*:** $$Q_i = \sum_j\left[A_i\cdot\Lambda_{ij}\cdot\sigma\cdot\left[(T_i)^4 - (T_j)^4\right]\right]$$

mat **Radiation** (R) is generated by $(A_i\cdot\Lambda_{ij})$
with definitions from **4**) and **5**)
for closed configurations

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**Angle factor properties** (to be verified for each new model) Fij=$\dfrac{\text{diffuse energy leaving Ai directly toward and intercepted by Aj}}{\text{total diffuse energy leaving Ai}}$

**For all *i* :** $\sum_j F_{ij} = 1$ and $F_{ii}$ not always equal to zero (for concave elements)

**For all *i,j* :** $A_i\cdot F_{ij} = A_j\cdot F_{ji}$ **Reciprocity rule** *(and not* $A_j\cdot F_{ij} = A_i\cdot F_{ji}$ *!)*

**Nota**

$$\sum_j F_{ij} = 1 \text{ and } \sum_j \delta_{ij} = 1 \text{ imply } \sum_j X_{ij} = 1 \text{ imply } \sum_j\left[(\text{mat X})^{-1}\right]_{ij} = 1 \text{ imply } \boxed{\sum_j \Lambda_{ij} = 0}$$

$$\text{mat}(A_i\cdot F_{ij}) \text{ symmetrical imply } \text{mat}(A_i\cdot\Lambda_{ij}) \text{ symmetrical}$$

Proof ! **Radiation matrix** (R) = $\mathrm{mat}\left(A_i \cdot \Lambda_{ij}\right)$ is symmetrical

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$$\Lambda_{ij} = \frac{\varepsilon_i}{1-\varepsilon_i}\cdot\left[\delta_{ij} - \left(X^{-1}\right)_{ij}\right]$$

It is sufficient to show that $\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i \cdot \left(X^{-1}\right)_{ij}\right]$ is symmetrical without the diagonal terms

$$\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i \cdot \left(X^{-1}\right)_{ij}\right] = \mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\right)\cdot\left(X^{-1}\right)$$

is symmetrical imply also for its inverse $\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i \cdot \left(X^{-1}\right)_{ij}\right]^{-1}$

$$\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i \cdot \left(X^{-1}\right)_{ij}\right]^{-1} = (X)\mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\right)^{-1} = (X)\mathrm{mat}\left(\delta_{ij}\cdot\frac{1-\varepsilon_i}{\varepsilon_i}\cdot\frac{1}{A_i}\right)^{-1} = \mathrm{mat}\left(X_{ij}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\cdot\frac{1}{A_j}\right)$$

$$X_{ij} = \frac{\delta_{ij} - \left(1-\varepsilon_i\right)\cdot F_{ij}}{\varepsilon_i}$$

It is sufficient to show that $\mathrm{mat}\left(\frac{1-\varepsilon_i}{\varepsilon_i}\cdot F_{ij}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\cdot\frac{1}{A_j}\right)$ is symmetrical without the diagonal terms

$$X2\ \mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot\frac{1}{A_i}\right)^{-1}$$

$\mathrm{mat}\left(\frac{F_{ij}}{A_j}\cdot\frac{1-\varepsilon_i}{\varepsilon_i}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\right)$ symmetrical if also for $\mathrm{mat}\left(\frac{F_{ij}}{A_j}\right)$

$\mathrm{mat}\left(\frac{F_{ij}}{A_j}\right)$ is symmetrical because the **Reciprocity rule** $\mathrm{mat}\left(A_i \cdot F_{ij}\right)$ symmetrical

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References :

**Radiation Heat Transfer**, Radiant interchange among diffusely emitting and diffusely reflecting surfaces, Interchange among Gray surfaces, E.M. **Sparrow**, R.D. **Cess**, *Hemisphere Publishing Corporation*

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**EcosimPro®**

The formulation of this memo has been implemented in EcosimPro simulation tool: but that is **much simpler**, because **only the main equations** are written in the CONTINUOUS part, without any matrix inversion and other algebra stuff.
(see the listing at right)

```
COMPONENT View_Factors
  (  INTEGER nports = 6 "Number of thermal ports connected by the component (-)"
  )  "Radiative heat transfer model among the nodes by means of the view factors, here for a room with 6 faces"
PORTS
  IN thermal (n = 1) tp_in[nports] CARDINALITY 0, 1 "Thermal inlet ports"
DATA
  REAL x=1 UNITS "m" "length along X axis"
  REAL y=4 UNITS "m" "length along Y axis"
  REAL z=2 UNITS "m" "length along Z axis"
  REAL e[nports] =1 UNITS "-" "Emissivity"
DECLS
  REAL Tk[nports] UNITS "K" "Absolute node temperature"
  REAL Q[nports] UNITS "W" "Net rate of radiated power: rule + when leaving the node i, - when heating the node i"
  REAL A[nports] UNITS "m^2" "Area"
  REAL H[nports] UNITS "W/m^2" "Incident heat power on i: rule + when arriving on the node i (i.e. + when leaving the other nodes j)"
  REAL B[nports] UNITS "W/m^2" "Radiosity : Emitted + reflected heat power: rule + when leaving the node i, - when heating the node i"
  REAL Emitted[nports] UNITS "W" "Emitted : heat power from i: rule + when leaving the node i, always positive"
  REAL VF[nports,nports] UNITS "-" "View factor for each coupling (-)"
  REAL ErrCumul UNITS "-" " near zero is ok; error check in the view factor function wrt Sum Fij and reciprocity rule AiFij=AjFji"
INIT
  ASSERT (nports > 0) FATAL "View_Factors component: Number of nodes must be at least 1"
  ASSERT (ErrCumul<1e-6) WARNING "View_Factors component ErrCumul"
  ViewFactorsRoomOrder162534( x,y,z,6, VF, A, ErrCumul ) --View factor function, face 6 is opposed to 1, etc.
CONTINUOUS
  EXPAND_BLOCK (i IN 1,nports)
      Q[i]=A[i]*Emitted[i]-A[i]*e[i]*H[i] -- e[i] is emission global factor = absorption too for grey bodies
      Emitted[i]=e[i]*STEFAN*Tk[i]**4
      H[i]=SUM(j IN 1,nports ;B[j]* VF[i,j])
      B[i]=Emitted[i]+ (1-e[i])*H[i] --(1-e[i]) is the reflection global factor.
      tp_in[i].q[1] =Q[i]
      Tk[i] = tp_in[i].Tk[1] -- Net temperatures in each node
  END EXPAND_BLOCK
END COMPONENT
```

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