Main hypothesis

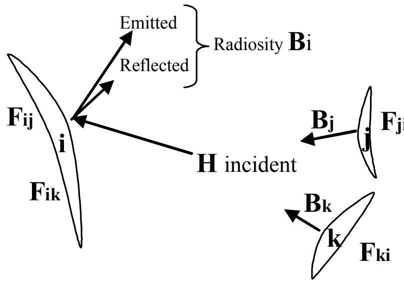
Uniform Radiosity

1) Uniform temperature surface of each element (node)
2)Grey surfaces, grey body : $\varepsilon_i = \alpha_i$ that is $\rho_i = 1 - \varepsilon_i = 1 - \alpha_i$
3)Diffusely radiated Reflexion
4)Diffusely radiated Emission
5)No Transmission

Nota:
Reflected ρ
Absorbed α
Emitted ε
Transmitted τ
and $\alpha+\rho+\tau=1$

Non intuitive notations F_{ij} the angular view factor from ***i*** toward ***j*** (see properties), A_i the surface (m²)

Net rate of heating (W) for the element ***i*** (rule Qi: positive when energy is leaving element **i**)



$$Q_i = \text{Emitted} - (\text{Incident} - \text{Reflected})(W)$$

$$Q_i = \text{Emitted} - \text{Absorbed from IncidentRadiation}(W) \qquad \rho_i = 1 - \varepsilon_i = 1 - \alpha_i$$

1) $$Q_i = A_i \cdot \varepsilon_i \cdot \sigma \cdot (T_i)^4 - A_i \cdot \alpha_i \cdot H_i$$

H_i: **Incident Radiant Heat flux** on the element ***i*** due to the other elements ***j*** (W/m²)
(rule Hi: positive when energy is arriving on **i** , thus positive as when leaving other **j**)

For pure black bodies $$H_i = \sum_j \left[\varepsilon_j \cdot \sigma \cdot (T_j)^4 \cdot A_j \cdot F_{ji} \cdot \frac{1}{A_i}\right]$$ With reciprocity rule, $$H_i = \sum_j \left[\varepsilon_j \cdot \sigma \cdot (T_j)^4 \cdot F_{ij}\right]$$

$$Q_i = A_i \cdot \sigma \cdot (T_i)^4 - A_i \cdot H_i$$

Black body emissive power =Heat power Emitted (W/m²) = $\sigma \cdot (T_j)^4$ with $\varepsilon_j = 1$. Because of $\sum_j F_{ij} = 1$, one have $(T_i)^4 = \sum_j \left[(T_i)^4 \cdot F_{ij}\right]$

Thus : **1bis)** $$Q_i = \sum_j \left[A_i \cdot F_{ij} \cdot \sigma \cdot \left[(T_i)^4 - (T_j)^4\right]\right]$$

For gray bodies, by analogy, the black body emissive power into $\mathbf{H}_i$ is replaced by the **Radiosity** B_j(W/m²) =Heat power **Emitted +Reflected**
(rule Bi: idem rule Hi)

2) $$H_i = \sum_j (B_j \cdot F_{ij}) \quad \text{with} \quad B_j = \varepsilon_j \cdot \sigma \cdot (T_j)^4 + \rho_j \cdot H_j$$ Note: this definition is implicit because B_j is also a function of B_k and itself B_j.

3) That gives also for the element *i* $$B_i = \varepsilon_i \cdot \sigma \cdot (T_i)^4 + \rho_i \cdot H_i$$ hence $$B_i = \varepsilon_i \cdot \sigma \cdot (T_i)^4 + (1 - \varepsilon_i) \cdot \left[\sum_j (B_j \cdot F_{ij})\right]$$ and $$H_i = \frac{\left[B_i - \varepsilon_i \cdot \sigma \cdot (T_i)^4\right]}{(1 - \varepsilon_i)}$$

4) $$\sum_j (X_{ij} \cdot B_j) = \sigma \cdot (T_i)^4 \quad \text{with} \quad X_{ij} = \frac{\delta_{ij} - (1 - \varepsilon_i) \cdot F_{ij}}{\varepsilon_i}$$ Note: Kronecker symbol : $\delta_{ii} = 1$; δ_{ij} =0 for $i \neq j$

With matrix notation, (B), (σT^4) being columns matrixes;
from **4**) view factors & emissivity, matrix (X), its inverse (X^{-1})

$$(X) \cdot (B) = (\sigma T^4) \text{ thus } (B) = (X^{-1}) \cdot (\sigma T^4) \text{ i.e. } B_i = \sum_j \left[(X^{-1})_{ij} \cdot \sigma (T_j)^4\right]$$

Equations *1)* and *3) for Hi* give $$Q_i = \frac{A_i \cdot \varepsilon_i}{1 - \varepsilon_i} \cdot \left[\sigma \cdot (T_i)^4 - B_i\right]$$ Bi is known when Tj are known

5) That gives for open configurations $$Q_i = \sum_j -\left[A_i \cdot \Lambda_{ij} \cdot \sigma \cdot (T_j)^4\right] \quad \text{with} \quad \Lambda_{ij} = \frac{\varepsilon_i}{1 - \varepsilon_i} \cdot \left[\delta_{ij} - (X^{-1})_{ij}\right]$$

---

6) **Finally,** because $\sum_j \Lambda_{ij} = 0$, **for all *i*:** $$Q_i = \sum_j \left[A_i \cdot \Lambda_{ij} \cdot \sigma \cdot \left[(T_i)^4 - (T_j)^4\right]\right]$$

mat **Radiation** (R) is generated by $(A_i \cdot \Lambda_{ij})$
with definitions from **4**) and **5**)
for closed configurations

Angle factor properties (to be verified for each new model) Fij=<u>diffuse energy leaving Ai directly toward and intercepted by Aj</u> / total diffuse energy leaving Ai

For all *i* : $\sum_j F_{ij} = 1$ and F_{ii} not always equal to zero (for concave elements)

For all *i,j* : $A_i \cdot F_{ij} = A_j \cdot F_{ji}$ **Reciprocity rule** *(and not* $A_j \cdot F_{ij} = A_i \cdot F_{ji}$ *!)*

Nota

$$\sum_j F_{ij} = 1 \text{ and } \sum_j \delta_{ij} = 1 \text{ imply } \sum_j X_{ij} = 1 \text{ imply } \sum_j \left[(\text{mat } X)^{-1}\right]_{ij} = 1 \text{ imply } \boxed{\sum_j \Lambda_{ij} = 0}$$

$$\text{mat}(A_i \cdot F_{ij}) \text{ symmetrical imply } \text{mat}(A_i \cdot \Lambda_{ij}) \text{ symmetrical}$$

Proof ! **Radiation matrix** (R) = $\mathrm{mat}\left(A_i \cdot \Lambda_{ij}\right)$ is symmetrical

---

$$\Lambda_{ij} = \frac{\varepsilon_i}{1-\varepsilon_i}\cdot\left[\delta_{ij} - \left(X^{-1}\right)_{ij}\right]$$

It is sufficient to show that $\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\cdot\left(X^{-1}\right)_{ij}\right]$ is symmetrical without the diagonal terms

$$\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\cdot\left(X^{-1}\right)_{ij}\right] = \mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\right)\cdot\left(X^{-1}\right)$$

is symmetrical imply also for its inverse $\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\cdot\left(X^{-1}\right)_{ij}\right]^{-1}$

$$\mathrm{mat}\left[\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\cdot\left(X^{-1}\right)_{ij}\right]^{-1} = (X)\,\mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot A_i\right)^{-1} = (X)\,\mathrm{mat}\left(\delta_{ij}\cdot\frac{1-\varepsilon_i}{\varepsilon_i}\cdot\frac{1}{A_i}\right)^{-1} = \mathrm{mat}\left(X_{ij}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\cdot\frac{1}{A_j}\right)$$

$$X_{ij} = \frac{\delta_{ij} - \left(1-\varepsilon_i\right)\cdot F_{ij}}{\varepsilon_i}$$

It is sufficient to show that $\mathrm{mat}\left(\frac{1-\varepsilon_i}{\varepsilon_i}\cdot F_{ij}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\cdot\frac{1}{A_j}\right)$ is symmetrical without the diagonal terms

$$X2\ \mathrm{mat}\left(\delta_{ij}\cdot\frac{\varepsilon_i}{1-\varepsilon_i}\cdot\frac{1}{A_i}\right)^{-1}$$

$\mathrm{mat}\left(\frac{F_{ij}}{A_j}\cdot\frac{1-\varepsilon_i}{\varepsilon_i}\cdot\frac{1-\varepsilon_j}{\varepsilon_j}\right)$ symmetrical if also for $\mathrm{mat}\left(\frac{F_{ij}}{A_j}\right)$

$\mathrm{mat}\left(\frac{F_{ij}}{A_j}\right)$ is symmetrical because the **Reciprocity rule** $\mathrm{mat}\left(A_i\cdot F_{ij}\right)$ symmetrical

---

References :

Radiation Heat Transfer, Radiant interchange among diffusely emitting and diffusely reflecting surfaces, Interchange among Gray surfaces, E.M. **Sparrow**, R.D. **Cess**, *Hemisphere Publishing Corporation*

www.KopooS.com

EcosimPro®

The formulation of this memo has been implemented in EcosimPro simulation tool: but that is **much simpler**, because **only the main equations** are written in the CONTINUOUS part, without any matrix inversion and other algebra stuff.
(see the listing at right)

```
COMPONENT View_Factors
  ( INTEGER nports = 6 "Number of thermal ports connected by the component (-)"
  ) "Radiative heat transfer model among the nodes by means of the view factors, here for a room with 6 faces"
PORTS
  IN thermal (n = 1) tp_in[nports] CARDINALITY 0, 1 "Thermal inlet ports"
DATA
  REAL x=1 UNITS "m" "length along X axis"
  REAL y=4 UNITS "m" "length along Y axis"
  REAL z=2 UNITS "m" "length along Z axis"
  REAL e[nports] =1 UNITS "-" "Emissivity"
DECLS
  REAL Tk[nports] UNITS "K" "Absolute node temperature"
  REAL Q[nports] UNITS "W" "Net rate of radiated power: rule + when leaving the node i, - when heating the node i"
  REAL A[nports] UNITS "m^2" "Area"
  REAL H[nports] UNITS "W/m^2" "Incident heat power on i: rule + when arriving on the node i (i.e. + when leaving the other nodes j)"
  REAL B[nports] UNITS "W/m^2" "Radiosity : Emitted + reflected heat power: rule + when leaving the node i, - when heating the node i"
  REAL Emitted[nports] UNITS "W" "Emitted : heat power from i: rule + when leaving the node i, always positive"
  REAL VF[nports,nports] UNITS "-" "View factor for each coupling (-)"
  REAL ErrCumul UNITS "-" " near zero is ok; error check in the view factor function wrt Sum Fij and reciprocity rule AiFij=AjFji"
INIT
  ASSERT (nports > 0) FATAL "View_Factors component: Number of nodes must be at least 1"
  ASSERT (ErrCumul<1e-6) WARNING "View_Factors component ErrCumul"
  ViewFactorsRoomOrder162534( x,y,z,6, VF, A, ErrCumul ) --View factor function, face 6 is opposed to 1, etc.
CONTINUOUS
  EXPAND_BLOCK (i IN 1,nports)
     Q[i]=A[i]*Emitted[i]-A[i]*e[i]*H[i] -- e[i] is emission global factor = absorption too for grey bodies
     Emitted[i]=e[i]*STEFAN*Tk[i]**4
     H[i]=SUM(j IN 1,nports ;B[j]* VF[i,j])
     B[i]=Emitted[i]+ (1-e[i])*H[i] --(1-e[i]) is the reflection global factor.
     tp_in[i].q[1] =Q[i]
     Tk[i] = tp_in[i].Tk[1] -- Net temperatures in each node
  END EXPAND_BLOCK
END COMPONENT
```

ed. 09