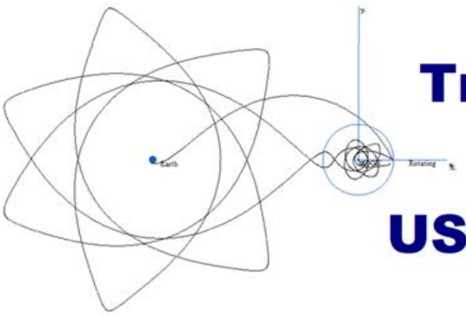
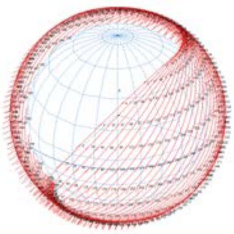
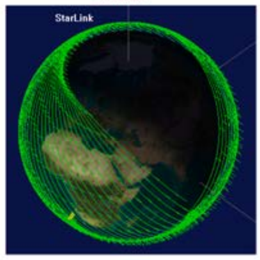
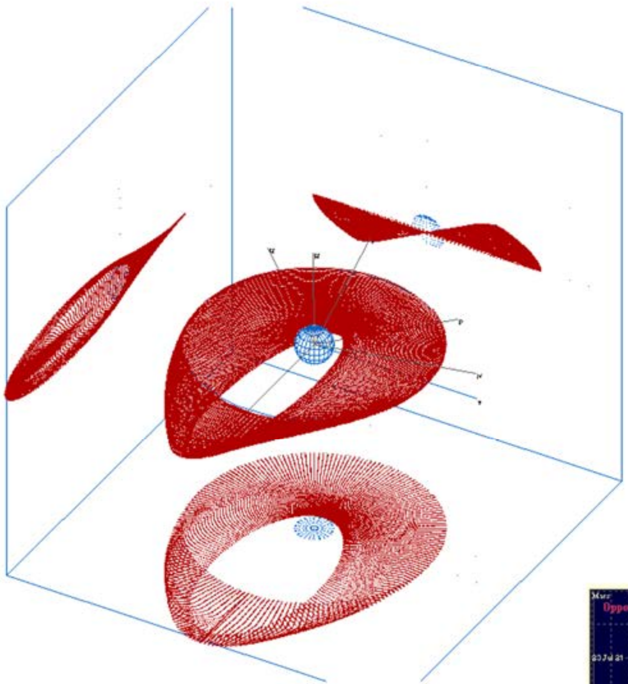


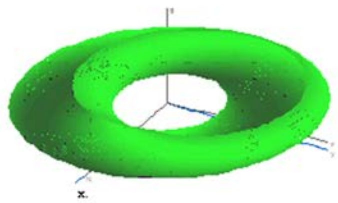
TriaXOrbitaL

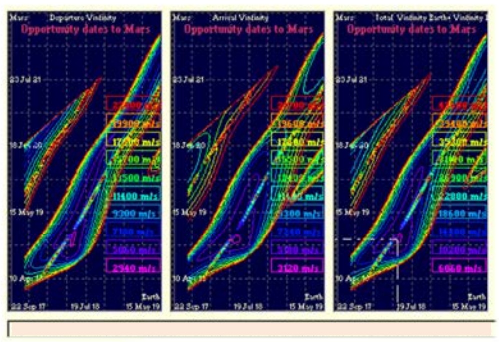
USER's Manual

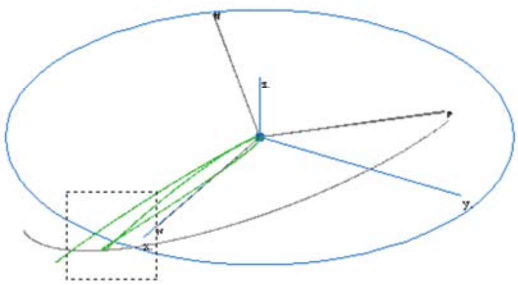


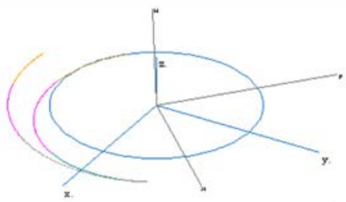












www.KopooS.com **Jan 2022**

USER's Manual TriaXOrbitaL

Congratulations and Welcome!!!

You are now the proud owner of an evaluation copy of TriaXOrbitaL, the fast, fun, and eas*y-to-*use space trajectory program & orbital perturbations program.

The use of the program is as simple as possible for engineers, students and autodidacts. The approach, when developing the program, has always been to make it accessible, as far as possible, to any person who is not a specialist of the Flight dynamic techniques.

TriaXOrbitaL is distributed as demonstration program to reach the widest possible audience, but it is still copyrighted material. You are granted permission to try out the program for 30 days, to see if you want to keep it and use it, and if you decide the program is worth a reasonable fee, and ***is free for the promotional release****. There are many easy ways to get in touch with us and many attractive incentives when you register.*
Also, you are encouraged to copy the entire shareware package and pass it around to anyone who may be interested, as long as you don't change any of the files and at the very most charge a small media and handling fee.

The program has been designed for computing and viewing dynamically in three dimensions the spacecraft trajectories with thrust periods and cruise (ballistic) periods. The program allows computing and viewing the secular orbital perturbations as well as their short period evolution. Interplanetary trajectories with or without thrust arcs are managed as well. The main rule for the use of the program being the traceability, each computation parameters is (or can be) saved in a customised database. The results of each computation are saved.

The recommended use of the program is to run a case from the database, to analyse the results dynamically during the computation process (and eventually generate a precise step by step file with the trajectory data and thrust orientation) and end the program before starting the computation of a new case.

The ascent compatibility in the program update and releases is one of its main features. Very old computation cases are compatible with the current program release. That allows also maintaining the accuracy and the bug free principle in a very efficiently manner.

Results of the computations using the program have been published in many papers and also used in patents (see references).

User's manual TriaXOrbitaL: Table of content

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2. Getting started

Before installing and/or using this product (tested on Windows 7, 8, 10) , please make sure you carefully read the copyright notice and agree on all its terms. The install procedure described here applies to the standard version of the TriaXOrbitaL program for the user who has the Administrator rights on his machine. For other user, for installing and using the version of the program anyway, please open the readme.txt file in the package and follows the instructions written in that file.

The package TriaXOrbital.zip contains all the files needed

- to install automatically the program on a hard disk (see same colour paragraph next),
- to start the use of the program with a predefined database of majors examples (see same colour §),,
- to fully use the program with the user defined data,
- to save updated records, to create the same number of new records as the number of examples,
- to open and analyse (preferably with Microsoft® Excel) the results files produced by the program.

The installation of the program is very similar to other software:

- Unzip the file TriaXOrbital.zip in a directory of your choice (or a temporary directory),
- Double click on the program setup.exe,
- Follows the instructions. ***If you do not have bought the program, the demo will be available for 30 days. The promotional release is unlimited. It also allows you to save some database records.***
- Special cases for user who are not administrator: please contact Support@kopoos.com.
- For user who has older windows systems : please contact us, it is possible to make special releases

To start the program, the user has to double click on the desktop icon programme named TriaXOrbital.exe

- Then the user shall follow the instruction for the login of TriaXOrbital and without password, click on **OK**. The main window is displayed.
- To start an example of the database, click on the button **Open Data**.
- A new window displays the last record of the **DataBase**. With the **scrollbar** you can change it, and then click on **Run** if the checkbox **Detail** is not checked,

  *else click on the button **Validation and Update** to see 4 next windows:*
  - The content of the data record is then displayed.
  - *A window displays the sub record **Configuration**. Click on Validation.*
  - *A window displays the sub record **Orbit**. Click on Validation.*
  - *A window displays the sub record **Manoeuvre**. Click on Validation.*
  - *A window, similar to the first one, displays the summary of the **DataBase record**. Click on Validation.*
  - *The main window is displayed again. Click on the button **Run**.*
- Follow the instruction for the login of the TriaXDraW *(the program graphic part)*, Click on **OK.**
  - A new window displays the **View** options for the graphics. Change it with the scrollbar and click on Validation.
  - Enjoy the three-dimensional visualisation in the window TriaX'
  - *Double click on a plot to change the view orientation or size, left click to Zoom, etc…*
  - *Select the main window and try the buttons Pause…Continue, Visual. Results, Visual Log, Close...*

For a first use of the program, that's all there is to it!

Now you are ready to enjoy looking at the trajectories with TriaXOrbitaL.

By the way, using the example, the future user is able to become familiar with some features of the program. It is highly recommended to run most of the already recorded examples in order to become very rapidly an experimented user, very efficient and productive.

Advanced user may require the reading of the further chapters to understand more on the content of the program, on some specific rules and specific definitions, and on the content of the files produced for further analyses of the trajectories and/or natural orbit perturbations.

The software has been designed to be as far as possible compatible without any mouse, however the mouse is recommended to enhance the Man Machine Interface. Without any mouse, the main keys are "Tab", "Alt" , "Alt + Tab", the arrows keys and the "space bar" or "Enter" .

Enjoy using TriaXOrbitaL,

3. Manoeuvre, thrust orientation, and thrust arcs.

In order to be as clear as possible the thrust orientation is an important characteristic of any software dealing with orbital manoeuvre. This is precisely defined here below.

- Manoeuvre

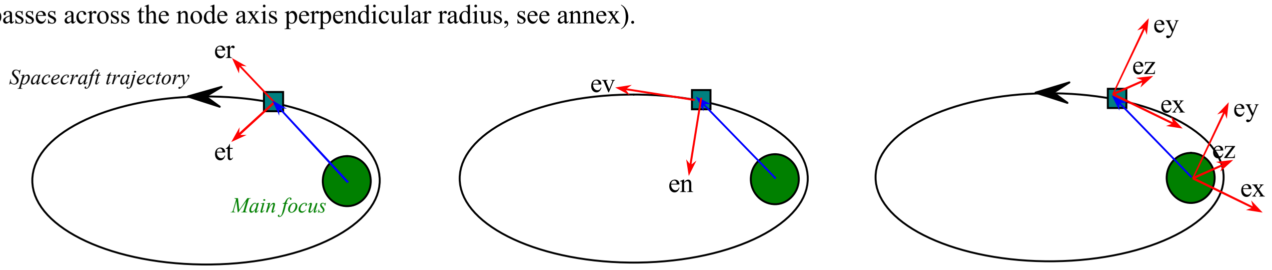
In the program, an orbital manoeuvre is defined as any trajectory computation case. However, most of the time, a manoeuvre includes some thrust arcs, or a large number of thrust arcs that are repeated from one orbit to the next one.

- Thrust vector orientation

Depending on the manoeuvre to be performed, the orientation of the thrust vector can be defined:

- With respect to the **polar orbital** local frame : radial, ortho-radial, angular momentum (er, et, eh),
- With respect to the **velocity orbital** local frame : velocity, ortho-velocity, angular momentum (ev, en, eh),
- With respect to an inertial (**Galilean**) orientation (with x for the vernal direction)
- With respect to the two body system rotating frame (synodic frame) : Not fully available.

Options are provided to perform optimised automatic orientation change (for example, when the spacecraft passes across the node axis perpendicular radius, see annex).



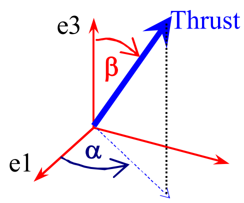
The **polar orbital** local frame.
The last base vector **eh** is out of the orbital plane.

The **velocity orbital** local frame.
The last base vector **eh** is out of the orbital plane.

The **Galilean inertial** frame,
with respect to the base vectors **ex**, **ey**, **ez**

The orientation specification is provided in TriaXOrbitaL through two angles:

- beta β, the angle of the thrust with respect to the third base vector (e3 : **eh** or **ez**), (sometime called as complement to declination)
- alpha α, the angle of the thrust perpendicular projection (*in dotted line*) with respect to the first base vector (e1 : **er** or **—en** or **ex**), (sometime called as right ascension).



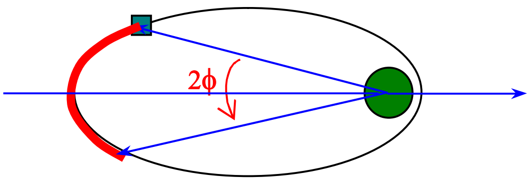
Note : for the velocity orbital local frame the angle α shall be given with respect to ***—en (minus en)*** *which plays generally, for circular orbits, a similar role as* ***er*** *in the polar orbital local frame.*

Note: some reference use ξ, the complement of β, that is ξ = 90°- β. (ξ is sometime called as declination)

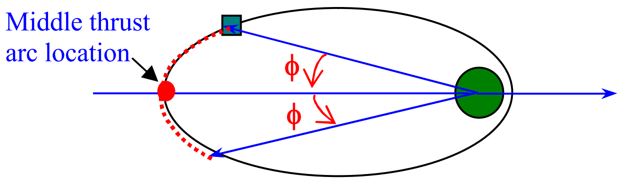
➢ Thrust arcs

A thrust arc is the part of the orbit where the thrust is provided (thrusters turned on). Depending on the spacecraft acceleration provided by the thrust, the thrust arcs may be very small or very large up to the entire orbit and repeated for many orbits: we call the last case a continuous orbit transfer.

The thrust arc is defined first with its **middle location** (perigee or apogee, …) on the local orbit and secondly with the **thrust arc amplitude** (half true anomaly angle or half eccentric anomaly angle). *For orbit transfer purpose, the locations of apogee or perigee does not change a lot, thus, actually, the thrust arc definition with reference to the middle point of the arc may allow to reduce the number of significant parameters, and thus to simplify any parametric study and trials.*

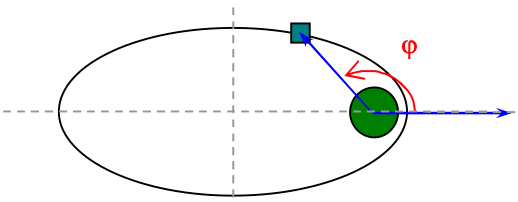


Example of **thrust arc** centred around the apogee with a true anomaly amplitude of **2ϕ.**

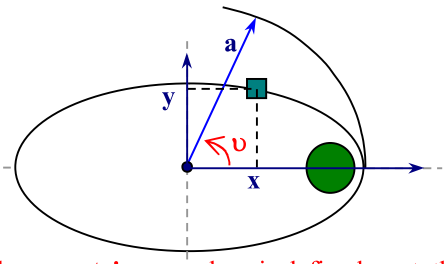


Specification of the same **thrust arc** in TriaXOrbitaL: half-thrust arc = ϕ , and the middle thrust arc location at apogee.

Note: true anomaly and eccentric anomaly are defined here below:

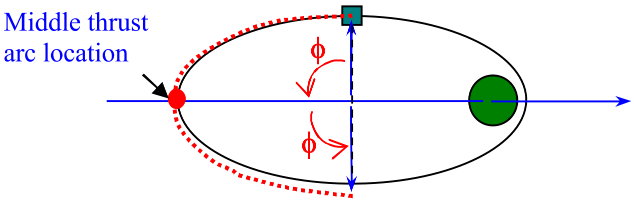


The **true** anomaly φ is defined w.r.t. the focus centre and from the perigee



The **eccentric** anomaly υ is defined w.r.t. the orbit centre, by the parametric equation of the ellipse : x=a.cos(υ) y=b.sin(υ),
with a=semi major axis, b= semi minor axis

Nota : a middle location at apogee with a half eccentric anomaly angle of 90° means a full *half* ellipse of thrust as sketched below.



Specification of the thrust arc in TriaXOrbitaL: half-thrust arc = ϕ as **eccentric anomaly**, and the middle thrust arc location at apogee.

Nota : a middle location at apogee with a 180° of half eccentric or true anomaly means a full orbit thrust (i.e. an elliptic spiral continuous orbit transfer strategy).

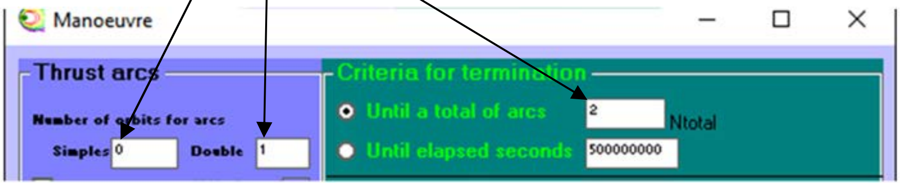
➢ Number of thrust arcs per orbit:

* one single thrust arc per orbit, the arc is called "simple arc" ,
* two thrust arcs per orbit, the arcs are called "double arcs".

In that last case, the thrust arcs are performed in orbital locations that are in opposition w.r.t. the focus (Middle thrust arc location at apogee and perigee for example).

- Thrust arcs repeatability: The program can be used for any kind of manoeuvre. Moreover, for the purpose of managing computations that are dealing with the feature of high specific impulse thrusters and thus with low thrust, **the thrust arcs can be widely repeated from one orbit to another orbit**.

The sequence of thrust arcs begins always with a number N_a1 *of simple arcs, then with a number* N_d *of double arcs and then a number* N_a2 *of simple arcs. Each number can be set to zero in order to skip the corresponding phase.*
Note: the tool includes inputs for N_a1 *,* N_d *and* N_{total} *So* $\mathbf{N_a2}$ *results from equation* $\mathbf{N_a2=N_{total}-N_a1-2*N_d}$.



Keeping in mind that the thrust orientation can be needed to be different for pulses at perigee and thrust at apogee, the logic of the configuration of thrusters used for thrust arcs N_a1 is always the configuration thrusters set n° 1 , while for N_a2 it is the configuration thrusters set n° 2. For double thrust arcs per orbit, it starts with configuration thrusters set n° 2 and continue with configuration thrusters set n° 1 and so on (*so, this logic allows a smooth continuation starting with not null* N_a1 *orbits*).

Example: For a orbit transfer starting with thrust arcs around perigee for N_a1 orbits, and then for N_d orbits with double thrust arcs (staring around apogee and second around perigee),

the thrusters **set n° 1** shall have the orientation required for a **perigee** thrust arc
and the thrusters **set n° 2** shall have the orientation needed for an **apogee** thrust arc.

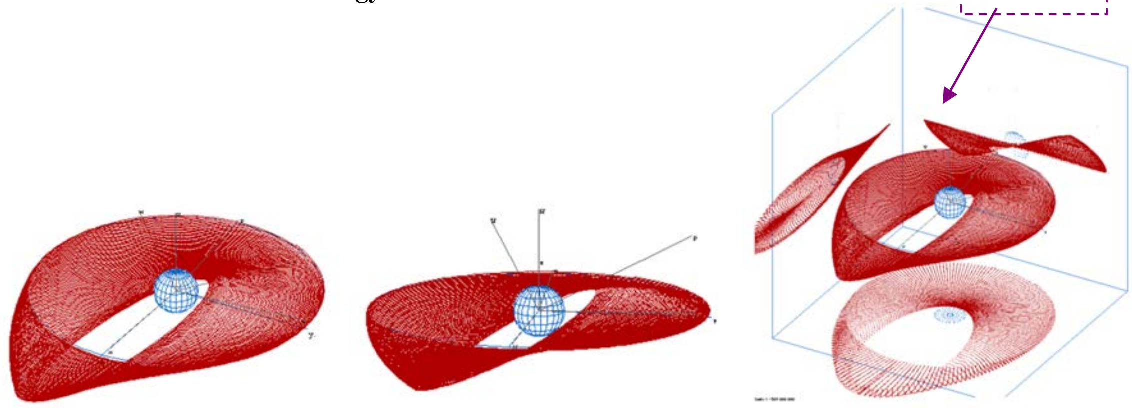
- Decision criteria for the end of the thrusting sequence: Criteria for the manoeuvre termination are build into the program: They can be based on the total number of arcs ($N_{total}=N_a1+2*N_d+N_a2$), on the total time, on the altitudes of perigee, apogee, eccentricity, etc…Note in any case, the total number of arcs is performed as far as possible if an other criteria is selected.

  Hence, please note that N_a1 , N_d and N_a2 *(deduced from* N_{total}) are specifications for **maximum number** of orbits because other criteria may be satisfied before the number of orbits reaches those numbers.
  A case with N_a1 = 10000 allow most of the time orbit transfers with a strategy of one simple single arc per orbit (up to a maximum of 10000 orbits *--the user can check afterward the real number of total arcs needed to perform its orbit transfer in the computation results, and increases it if needed--)*

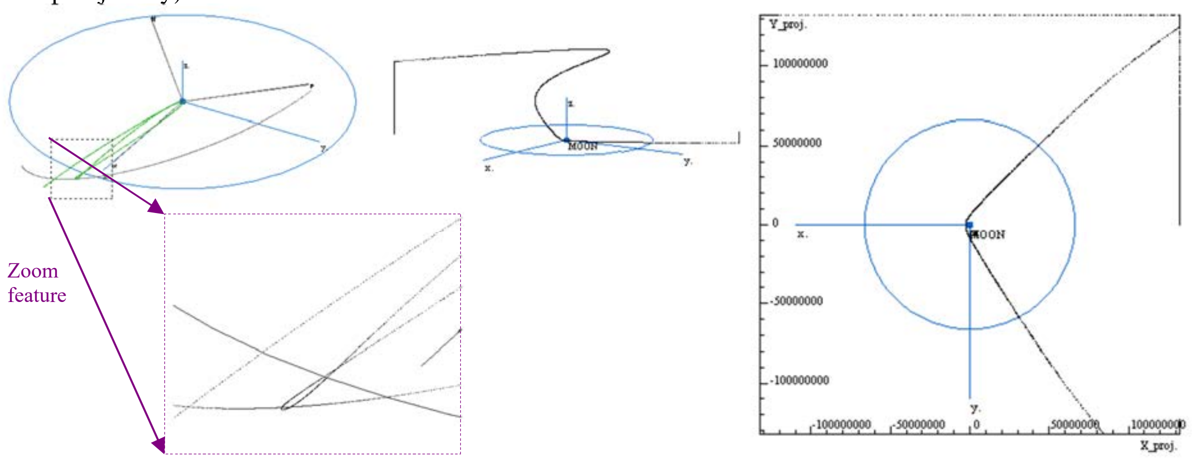
4. Some examples of Manoeuvres

The following examples are part of those provided with the genuine database included in the program package.

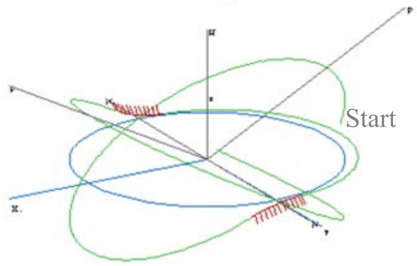
Continuous orbit transfer strategy from GTO inclined 28° to GEO : standard views and cubic view



Moon flyby standard views and zoom view of the flyby event and free Earth return (ref., Kaplan Apollo 11 back-up trajectory)



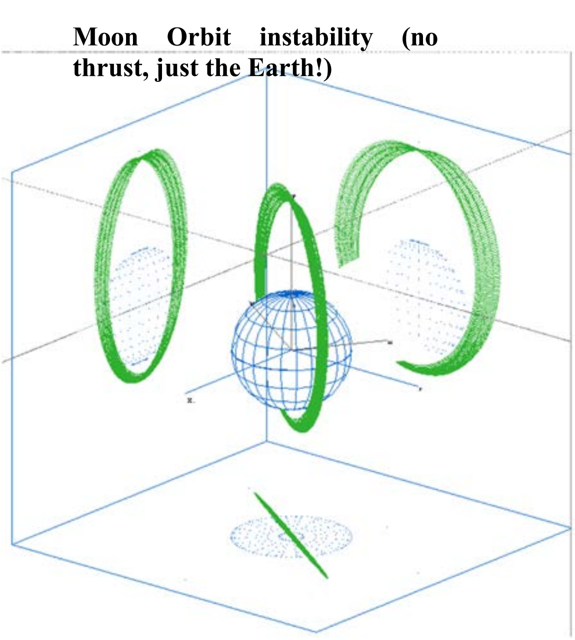
North South station keeping from anti-Earth side (See also in annex the major GEO perturbations with Moon, Sun and Earth polar oblateness)



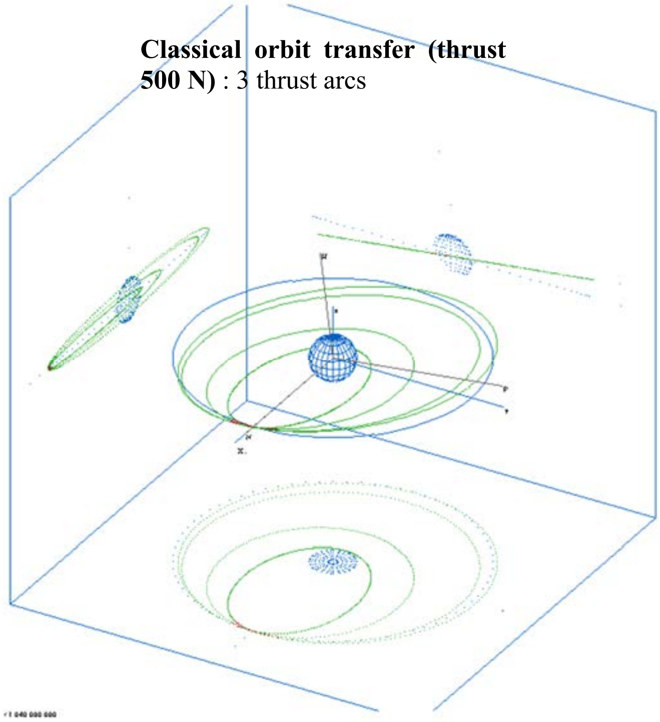
- Examples using the **Inclination Zoom feature** (Z coordinate 10000 times larger) in combination with the **Radius Range feature** set to only +-20 km with respect to GEO (blue circle here below).
- From GEO to the frame centre, there is thus 20 km. The user can see directly on the graph the slight orbit shift after the thrust arc which is removed after the second thrust arc 12 hours later.
- *Note: because the radius range do not includes any part of the Earth surface, the centre of the frame do not displays the Planet's sphere like in the other plots.*

Other examples

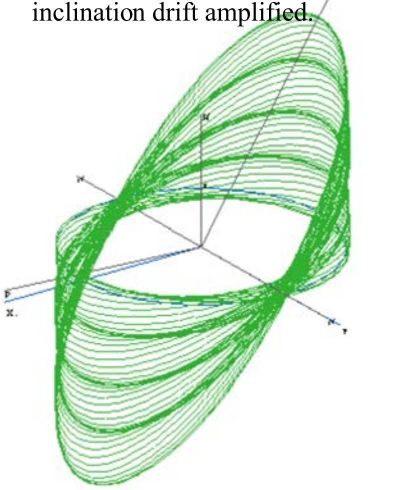
Moon Orbit instability (no thrust, just the Earth!)



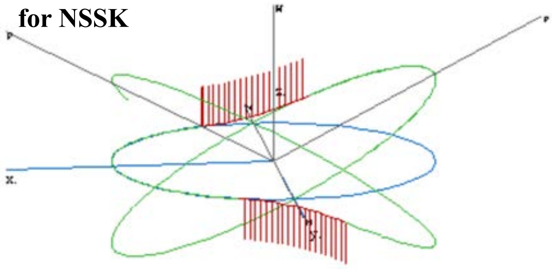
Classical orbit transfer (thrust 500 N) : 3 thrust arcs



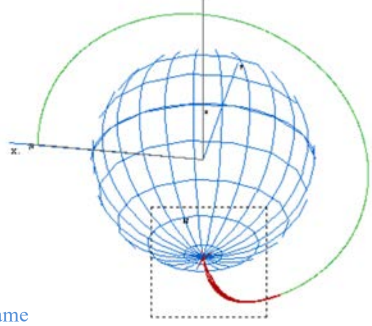
GEO perturbed by the Moon 56 days : the plot shows the secular inclination drift amplified.



Exact North and South thrust for NSSK



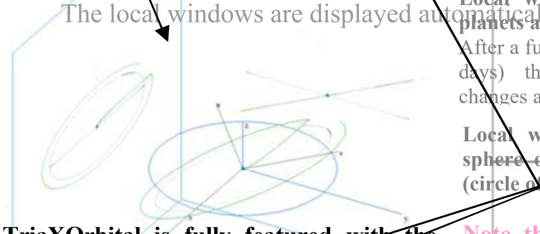
Soft landing on the Moon South pole. The criterion ApoAlt < 0 is used to terminate the computations. The user can adjust the initial altitude or the thrust to reach the surface with a velocity of 0 m/s

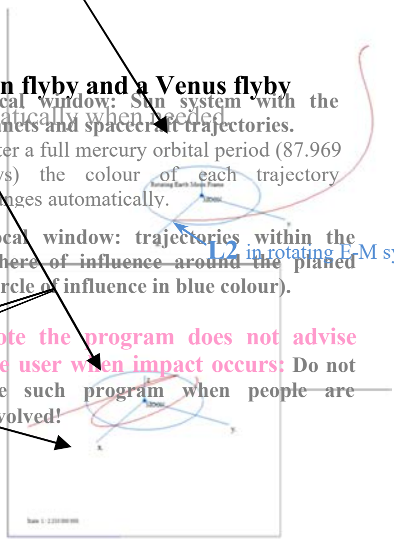


Typical feature used for reaching E-M L2 with 3 plots Earth ECI; local Moon in Rotating Earth-Moon and local Moon in ECI

Interplanetary trajectory with a Moon flyby and a Venus flyby

The local windows are displayed automatically when needed.





Local window: Sun system with the planets and spacecraft trajectories.

After a full mercury orbital period (87.969 days) the colour of each trajectory changes automatically.

Local window: trajectories within the sphere of influence around the planet (circle of influence in blue colour).

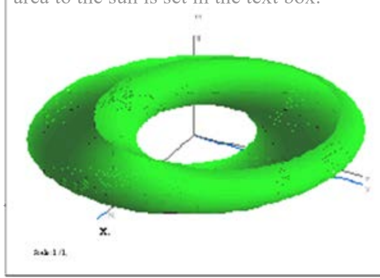
L2 in rotating E-M synodic frame

Note the program does not advise the user when impact occurs: Do not use such program when people are involved!

Note: TriaXOrbital is fully featured with the possibility to make the Moon orbit circular. This enable all simulation in the so called CR3BP (circular restricted 3 body problem).

In the right plot "rotating Earth-Moon" the trajectory seems to stop at Lagrangian point L2: this is not an error, moreover that is a **powerful check of the accuracy of the computations performed**. No other tool can provide such accuracy without solving the problem in the synodic frame.

Sun pressure perturbation of a Geosynchronous orbit. The check box "sunpressure" is checked, the front area to the sun is set in the text box.



List of the examples included into the original software package

During the installation of the software, two subdirectories are created under the installation directory. The name of the subdirectories is \data\ and \xls_doc\.

*Note: Under Windows 8, the true files generated or updated are however really stored in the program directory under the so called **Virtual Store** located at C:\Users\...\AppData\Local\VirtualStore\...but no worries, it works fine as explained here below. You may probably make a shortcut for accessing fast to the program files in the VirtualStore location.*

The installation directory is by default "C:\Program Files (x86)\TriaXOrbitaL\", in that case, the subdirectories are the following:

C:\Program Files\ TriaXOrbitaL\ data \
C:\Program Files\ TriaXOrbitaL\ xls_doc \.

- The subdirectory \data\ contents the database.
- The subdirectory \xls_doc\ will contents the results file (document and text file) that can be analysed by the user with any office tool (Excel or Word from Microsoft® by example).

The original database located in the subdirectory \data\ is composed of a number of files having the extension ".set".
The next table presents the records that the original database contents. A wide variety of examples are thus available to the users for reference.

n°record	Dataset	Title	TimeStamp
1	1.1	North and South thrust for NSSK	mardi, 1 janvier, 1991 01:01:02
2	1.3	North South Station Keeping from anti-Earth side	mardi, 1 janvier, 1991 01:01:02
3	1.01	GEO Moon perturbation, 56 days	mardi, 1 janvier, 1991 01:01:02
4	2	Classical orbit transfer (thrust 500 N)	mardi, 1 janvier, 1991 01:01:02
5	3	Continuous orbit transfer strategy from super GTO inclined 28°	mardi, 1 janvier, 1991 01:01:02
6	12	Interplanetary trajectory with a Moon flyby and a Venus flyby	mardi, 1 janvier, 1991 01:01:02
7	11	Moon orbit instability (no thrust, just the Earth!)	mardi, 1 janvier, 1991 01:01:02
8	11.2	Moon soft landing with chemical propulsion	mardi, 1 janvier, 1991 01:01:02
9	11.3	Moon South pole soft landing (chemical propulsion)	mardi, 1 janvier, 1991 01:01:02
10	10.1	Free return trajectory, Apollo 11 back-up (ref. Kaplan)	mardi, 1 janvier, 1991 01:01:02
11	12	Interplanetary trajectory Earth to Mars 2013	mardi, 1 janvier, 1991 01:01:06
12	10.2	LEO to E-M L 2 Lagrange point with impulse dV when at L2.	mardi, 1 janvier, 1991 01:01:09

….

5. Results files.

For the traceability of the computations performed, a files system is produced and managed by the program. The content and the meaning of the files is precisely defined in annex.

The programme produces two kinds of results files, saved in the user subdirectory "…**xls_doc**\":
(under Windows 8.1, the location is in the virtual store and within the program it is transparently managed, but for archives, the exact location is:"**C:\Users\...\AppData\Local\VirtualStore\Program Files (x86)\TriaXOrbitaL\xls_doc**\" where "..." stand for the user's name)

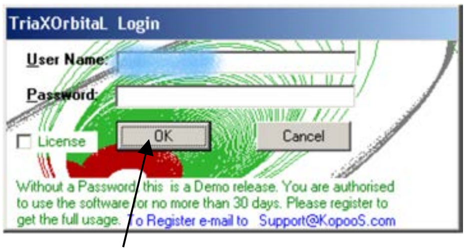
- An **events summary file** called "**Res_Tmp.xls**" in the subdirectory "**xls_doc**\" of the program directory. This file contains, in text form --i.e. ASCII--, all the orbital data and parameters settings for each events, i.e. each time the thrusters are activated like switched ON or switched OFF (or at each full orbit when the continuous thrusting strategy is selected). *Note: the file is renewed for each new run of the program.*

- A **detailed text file** called "**TriaxPol.xls**" in the subdirectory "**xls_doc**\" of the program directory. This file contains all the results of the computation steps, at each time step. This file may be very large, that is now no more an issue for its storage. *Note: the file "**TriaxPol.xls**" is however renewed for each new run of the programme. For huge amount of data a special mechanism allow to select data modulo 2 or 4 or etc. each time the size of the file double wrt an initial size of 5 megabytes*

Note: the text file with the extension "**.xls**" can be opened directly with Microsoft® Excel for further analysis with two dimensional plots (or also a three dimensional plot with an additional program, like **TriaXExcelPro** available on the web at www.KopooS.com which is fully compliant to show up to a million of points in Live 3DD with already written the specification for using it after the data distribution feature of Excel where it is recommended to use the space as delimiter during the conversion text to column process of Excel.

6. Menu features

The use of the program can start with the login checks (**phase skipped for the promotional release**):

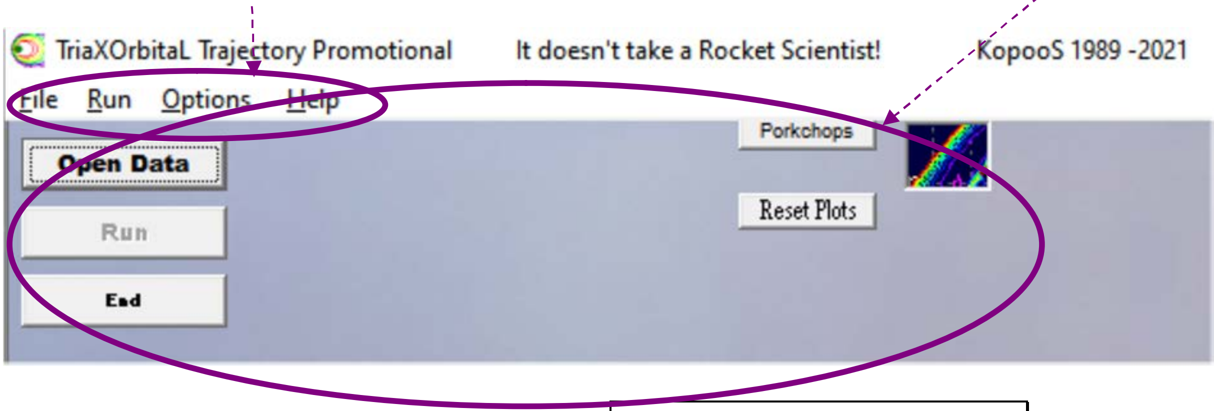


The non registered user shall click on OK to use the programme for a limited period of time.

The program displays, after the login, a main window named TriaXOrbitaL Trajectory. This window allows the user to select the main functions. The user can follow two ways: use of the menu bar or use of the command buttons. Only few of high level commands are available for an easier use.

List of all high level commands

The list of all the high level commands is provided here below: two kinds of commands are possible, the buttons commands and the classical menu bar.



Buttons commands in the window TriaXOrbitaL Trajectory.

Open Data : command that allows to select the database records (and to generate new record). Display the windows of the mandatory settings for the computation.

Run: command to compute the orbit manoeuvre. When the data have not been set, this command is hidden (greyed).

Constellation: optional command to view some of the static constellation orbits.

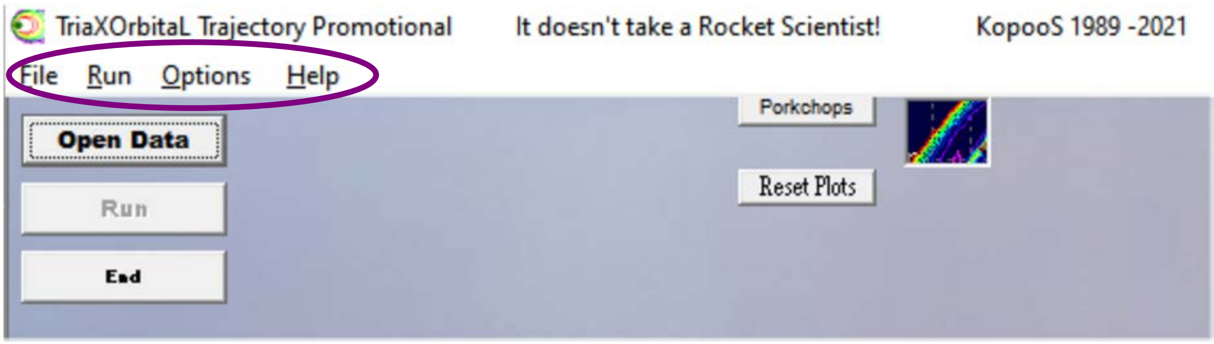
Sun System: optional command to view the sun system evolution after integration computations. Not available.

Porkchops: direct command to display the opportunity dates between planets.

ResetPlots: the current plots are cleared ; the trajectory is plotted from the current time

End: command to Exit the programme.

Menu bar commands in the window TriaXOrbitaL Trajectory:



File : Basic functions

Open Data : command that allows to select one of the database records (and to generate new record). Display the windows of the mandatory settings for the computation.

End: terminate the programme.

Run

Run: command to compute the orbit manoeuvre. Until the data have not been set, this command is hidden (greyed)..

Options

Results in Text file: allow the user to generate a clear detailed text file of the previous computation. The size of the file generated may be huge.

Add-on 3D from Excel : Not available

Constellations : optional command to view some of the constellation orbits.

Sun System: Not available

Porkchops: optional command to display the opportunity dates between planets.

Configuration: display of the settings of the programme (mainly, location of the directory for the default installed database and the results files).

Help

Help summary: display a summary window containing the basic functions help

A Propos de TriaX : displays the program support address…

Demo : Not available. See the examples in the database…

7. Detailed windows & commands after selection of a high level command.

The detailed windows & commands after selection of a high level command are provided here below.

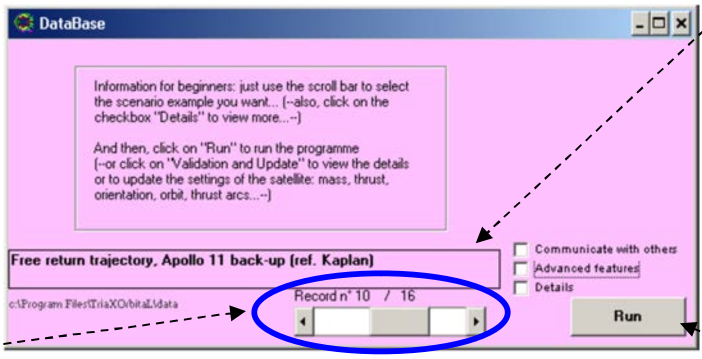
Windows & commands displayed after command Open Data

Forewords

The windows displayed after a click on the command button Open Data have all the basic functions for the database management: Scrollbar for the records selection, Save command in conjunction with the replace checkbox to generate a new record or the replace the existing one, Validation… : command that allows to view the next window.

Window DataBase available with Open Data command

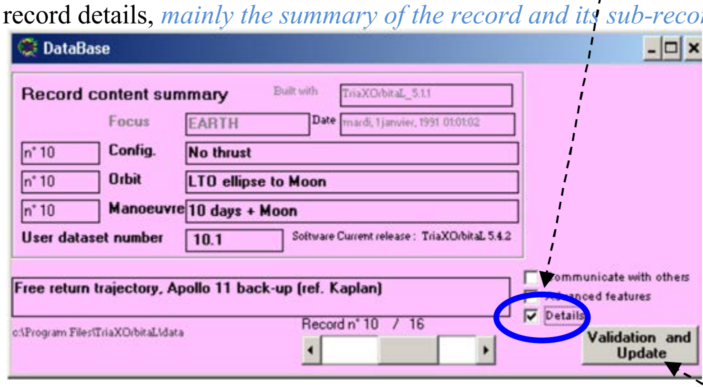
A database record is defined with sub-records and a unique record number: For its convenience, the user can add a record title and a user dataset number. The software release and the time stamp (*i.e. time of the database record save*) are also automatically printed into the database record when saved.
Each sub-record includes its own unique record number, and its title. *The data they contain are set in the further windows. The window* DataBase *displays first a very short message for beginners and the title of the current database record scenario.*



Use the scrollbar to select the database record you want to run and then click on the button Run

- **Detailed Mode**

If the user moves the mouse over the checkbox Details then the programme go into the detailed mode. The record details, *mainly the summary of the record and its sub-records content,* are displayed.



Summary of the record and its sub-records content

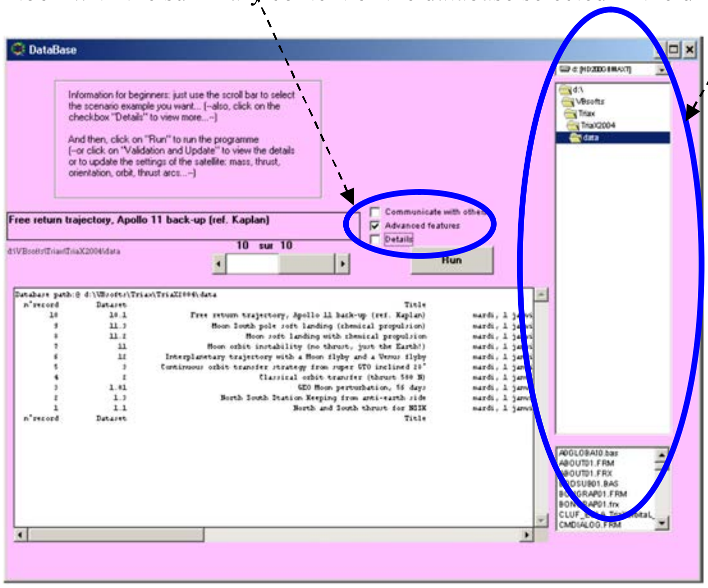
- Focus: this important information, selected in the Orbit window is displayed,
- Config.: sub-record number and title,
- Orbit: sub-record number and title,
- Manoeuvre: sub-record number and title,

Moreover, in this detailed mode the user can, in the further windows, visualise and /or update the settings of the scenario… *The label of the button* *Run* *is thus renamed to* *Validation and Update*.

For enabling each sub-record for being reviewed or updated, press Validation and Update. For ending the detailed mode, uncheck the checkbox Details.

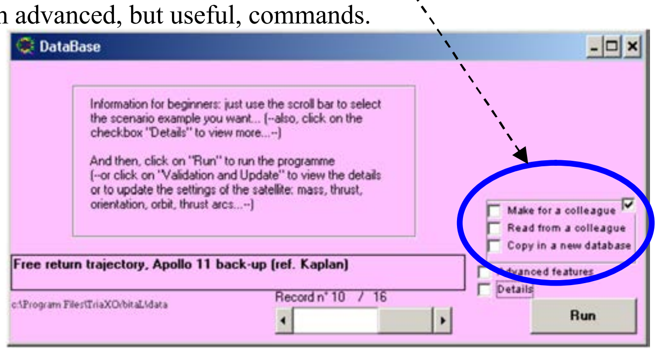
Advanced features

If the user clicks the checkbox Advanced features then the programme displays a larger window with a directory tree and a textbox with the summary content of the database selected in the directory tree.

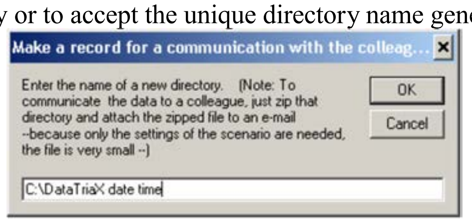


Communication with others

If the user clicks the checkbox Communication with others then the programme displays three new checkboxes that perform advanced, but useful, commands.



The checkbox Make for a colleague allows, when checked, creating from the current record number, a single record database that can be sent to a colleague for further discussion or analyses. The user is prompted to write or to modify or to accept the unique directory name generated by the programme.



Once performed, the user shall zip that directory and all the files belonging to it and shall send the zipped file to the colleague.

Note: if the directory name already exists with a database, the current record is simply added to that database.

The checkbox Read from a colleague allows, when checked, to use the database (after unzip) creating by the colleague.

Note: Under ***Windows 8*** *it is probably better to select other drive than C for cleanliness reasons.*

Window DataBase: tree of available features

Scrollbar: records selection

Checkboxes

Details : this checkbox command allows the user to go into the Detailed mode, to view the details of each record, and to update the settings in the next windows.

Advanced features: this checkbox commands the opening of an extended window showing the current database content (and allow to select with a mouse double click other record) and also a right side window tree in order to select other directory for the current data base. ***Please note that that the new selected path is saved in the parameter of the programme for the next runs.***

Communication with others

Make for a colleague: This checkbox command allows generating a new database with the current record for being able to transmit it to a colleague in a single zipped file.

Read from a colleague: This checkbox command allows the reading of an unzipped database transmitted via e-mail by a colleague.

Copy in a new database: this checkbox commands allows to generate a set of records in a new directory of database from the current data base

Copy: this commands allows to generate a new set of records in a new directory from the current data base

Run: ***command that allows running the trajectory without review of the data.***

Validation and Update***: command in*** **Detailed mode** ***that allows viewing the next window to display / modify / update the data.***

Copy of the previous: this command appears only at the second view of that window. It allows the user to generate a copy of the previously viewed record in order to make slight changes in a new record All the sub-records are also created in the same time.

Note: when you start the program with a new empty database, the window display no information (but a demo default case is already set in the code), click on validation to set or update the first record of the new database.

Window Configuration , available with Open Data , command Validation...

The configuration window includes the settings for the satellite and the location of the thrusters. The concept of thrusters set is used: that is a number of thrusters used simultaneously to produce one thrust vector (the resultant thrust vector being not necessarily the arithmetic sum of each unit thrust).

The configuration allows to set the following information: Satellite Mass, Specific Impulse, unit thrust and number of thrusters per thrusters set (and the total resultant thrust vector efficiency in the case of more than one thrusters per thrusters set). *The intensity of the thrust that can be plotted in the graphic viewer can be modified in the text cell of the amplification factor.*

Thrusters Orientation

The thrusters orientation is defined for a maximum of two set of thrusters: thrusters set n° 1 and thrusters set n°2. The orientation of each thrusters set can be defined, using the options, with respect to 3 main frames: Inertial absolute (i.e. Galilean frame), Polar orbital or Orbital velocity, thanks to the two angles of right ascension and complement to declination (α , β) as shown in the interactive sketch.

Example: thrusters set n° 1 have appropriate orientation for perigee thrust arcs, while thrusters set n° 2 have appropriate orientation for apogee thrust arcs, and the user shall set in window Manoeuve the appropriates values for single arcs per orbits and for the apside selected for the first thrust arc.

The user shall check in the results if the thrusters for perigee and apogee are the appropriate ones with their specific value α, β *(putting in window Manoeuvre: for Simples (N_a1) =1 or 0 may reverse the sequence of use, putting a "First thrust Arc Centered on" around perigee or apogee may reverse too. Hence, the user may have to play with those two settings for getting what he wants really for using the thrusters set n°1 for which apside.*

Integration step

The configuration allows setting the value of time step for the trajectory integration. However, the time step in times units itself does not represent really an understandable quantity. Instead, the specification of the time step in units of true anomaly (in degrees) is by far better manageable: at apogee or at perigee, the integration process produce about the same integration accuracy (or error) for a given true anomaly time step. *Note: If the step is set to zero (degrees), then an automatic process compute the optimised time step for each integration step. The goal is to get an integration error on the orbital radius (including 64 times the error on the velocity multiplied by the time step) not higher than the value of Epsilon (in meters). See* "Annex 3: Integration accuracy".

Additional Features

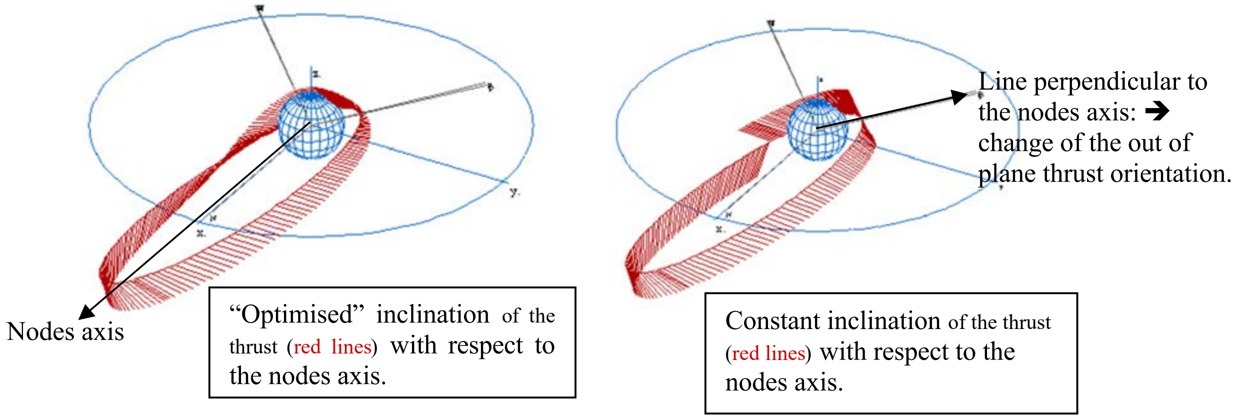
Two additional features can be set in that window: Options for the particular use of the thrusters sets are provided, and an advanced feature for the inclination optimisation process is provided when checking the checkbox Expertise inclination.

Particular use of the thrusters sets: in some case the two sets of thrusters will be used simultaneously for the whole manoeuvre: the user will select the corresponding option in the Mode zone. For other cases, the user may want to use alternatively the thrusters set n°1 then the thrusters set n° 2 then again the N°1 etc…

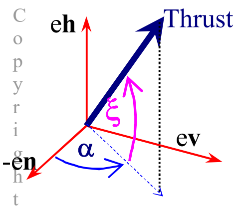
Advanced feature for the inclination optimisation process provides three possibilities:

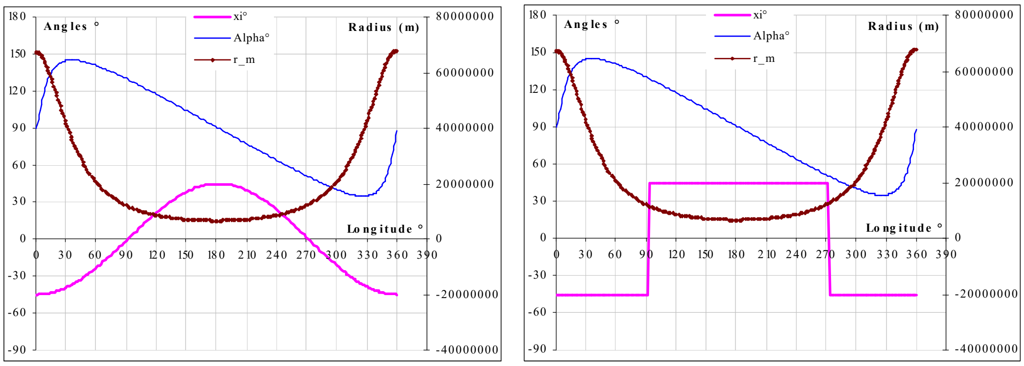
- Optimised inclination with respect to the nodes axis (see plots here below):
  The out of plane component is first governed by the specified angle β *or its complement to 90°* ξ, but in this expertise mode, that angle follows a cosinusoidal law of the argument of latitude, that is $\xi = \xi_0 * \cos$ (angle from the ascending node) where ξ_0 corresponds to the specified value $\beta_0=90°-\xi_0$. This is considered by some experts as a good starting point for an optimal law,
  The in-plane component of the thrust (in the orbital plane) is governed by the angle α but, its module is thus adjusted in order that the module of the **total resultant thrust remains constant** at its specified value.
- Constant inclination with respect to the nodes axis (see plots here below):
  Similarly as before, the angle ξ vary but with only two values. $\xi = \xi_0$ * **sign** (cos(angle from the ascending node))) where ξ_0 corresponds to the specified value $\beta_0=90°-\xi_0$. This strategy may also be considered as quasi-optimal while the **total resultant thrust remains constant** at its specified value.
- A thrust orientation which follows the spin of the satellite.

Example of thrust vector orientation using the "Optimised inclination with respect to the nodes axis" and the "Constant inclination with respect to the nodes axis" :



The corresponding orientation angles of thrust vector right, ascension and declination (α , ξ) from the examples using the ""Optimised" inclination with respect to the nodes axis" and the "Constant inclination with respect to the nodes axis" are plotted on Excel using the TriaXOrbitaL output file "TriaxPol.xls" (the in-plane component is, in those examples, collinear to the velocity, ξ_0=45°).





The three possibilities can be selected when the corresponding check boxes are checked (see annex).

Management of the out-of-plane orientation of the thrust in inertial absolute frame

In addition to the above mentioned expertise checkbox, a checkbox allows the management of the inclination changes when an inertial absolute frame is selected (or used via the criteria, see here below § window manoeuvre).

The check box is named Inertial absolute frame: Out of plane Inertial % orbital plane. This is coming from the fact that for the orientation defined in the polar orbital frame or in the velocity orbital frame, the out of plane angle is defined with respect to the orbital plane, while in an inertial absolute frame, the similar angle is defined and specified with respect to the orbital plane plus the initial orbital plane inclination, which give a total angle with respect to the equatorial plane.

In order to get similar understanding of the inclination change manoeuvre when using the expertise feature, as with the other orbital frames –polar or velocity--, the checkbox:

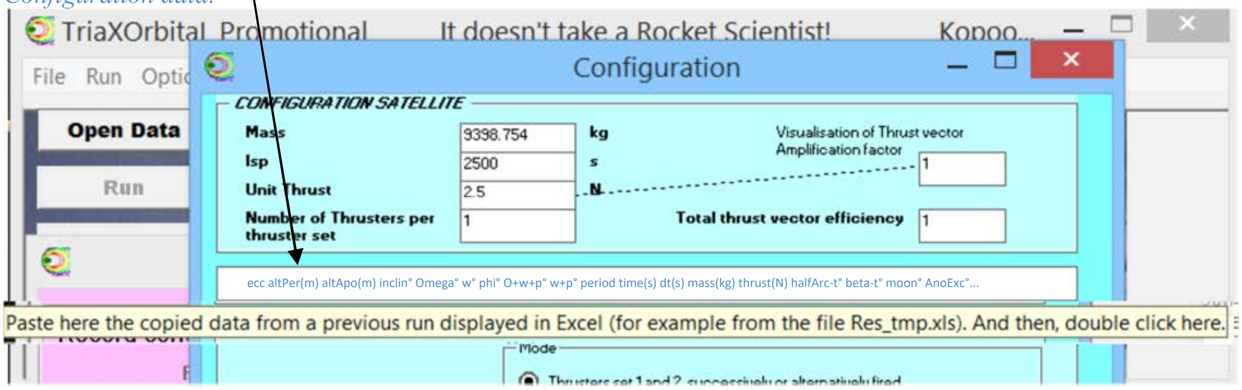
Inertial absolute frame: Out of plane Inertial % orbital plane,

available when an inertial absolute frame is selected, allows to define and specify the out of plane angle also with respect to the orbital plane only. This specification remains valid all over the whole manoeuvre. This specification is highly recommended, so that the out of plane angle do not depends on the initial orbital inclination.

One use the scrollbar to select the sub-record to set (or to modify): its unique record number is displayed above the scrollbar. Then the user can modify the settings or change the sub-record title. Then the user can Save it with the new time stamp (or replace it if the checkbox Replace is checked) … or/and then press Validation.

Note if the checkbox Replace is not checked, the Save command generate a new unique sub-record number. If the sub-directory "/Data" is read-only protected, the save command abort, and a warning is added to the title.

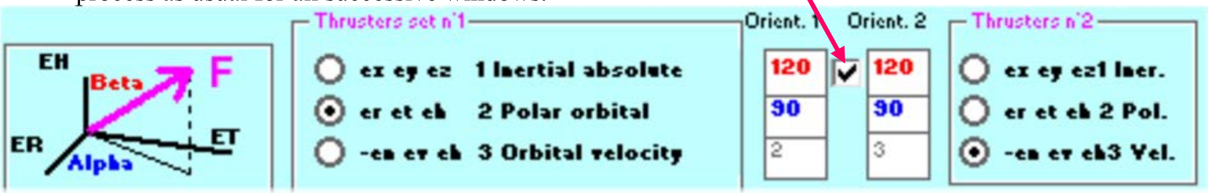
__Data from previous runs:__ the user can copy paste results from previous run in the text box area that become visible with the mouse . Then with a double click on that text area some data are automatically used to update Configuration data.



__Automatic adjustment of the Beta angle for reaching a zero inclination at the end of the Orbit Transfer__

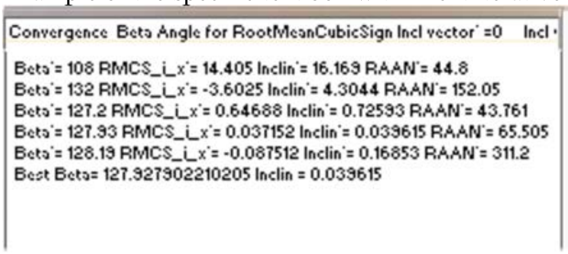
The Beta angle is the Out Of Plane angle of the thrust vector for Polar orbital or Orbital velocity frames. **It is needed to adjust its value** for reaching, at the end of the Orbit Transfer, the inclination value required.

For reaching a <u>zero inclination</u> at end of the Orbit Transfer, it is possible to do it automatically: one has to click on the check box between the two Beta values and continue the VALIDATION process as usual for all successive windows.



Once the RUN command is clicked, a **first phase** is performed very fast (and with few outputs) **to adjust the value by a Brent iterative method**. A specific text box follows the **convergence process**. Once the best value of Beta has been found, the full plots continue with the best Beta for reaching a zero inclination.

Example of the specific text box with Brent iterative method, here in 5 steps for full convergence.


Convergence Beta Angle for RootMeanCubicSign Incl vector' =0 Incl

Beta'= 108 RMCS_i_x'= 14.405 Inclin'= 16.169 RAAN'= 44.8
Beta'= 132 RMCS_i_x'= -3.6025 Inclin'= 4.3044 RAAN'= 152.05
Beta'= 127.2 RMCS_i_x'= 0.64688 Inclin'= 0.72593 RAAN'= 43.761
Beta'= 127.93 RMCS_i_x'= 0.037152 Inclin'= 0.039615 RAAN'= 65.505
Beta'= 128.19 RMCS_i_x'= -0.087512 Inclin'= 0.16853 RAAN'= 311.2
Best Beta= 127.927902210205 Inclin = 0.039615

Because the inclination angle is always >=0, the inclination is traced internally for finding its minimum and a pseudo algebraic inclination is output for finding the zero by Brent.

Window Configuration: tree of the available features

- **Scrollbar: records selection**
- **Checkboxes**
  - **Inertial absolute frame: this command allows the out of plane thrust component to be oriented as inertial with respect to the orbital plane.**
  - **Expertise inclination: this commands the view of a small window having the following checkboxes:**
    - **Optimised Inclination wrt Nodes**
    - **Inclination Constant wrt Nodes**
    - **Thrust Orientation following Spin**
  - **Replace: records**
- **Save: records save**
- **Validation*: command that allows to view the next window***

Window Orbit , available with Open Data , command Validation...

The Orbit window includes the settings for the satellite orbit as well as those dealing with the visualisation of the orbital manoeuvre.

The initial orbit parameters are:

- Altitude of apogee
- Altitude of perigee
- Inclination (abbreviated **i°**, where symbol **"°"** means that the unit is the degree)
- Right ascension (named usually Ω abbreviated **Om°**)
- Perigee argument (named usually ω abbreviated **w°**)
- Composite angle from the node to the current location (named usually $\theta = \omega + \varphi$ abbreviated **th°**)
- The starting focus name to be selected in the scrolling list: Earth, Moon, Sun, Mercury, Venus, Mars, Jupiter, Saturn.

In order to help the user for repetitive tasks, a set of predefined options of "useful values" are also available. The user may adapt the data for its own use (and all can be saved in the sub- record).

In case of a hyperbola starting orbit, check the checkbox Hyperbola and the Infinite velocity will be asked instead of the Apogee altitude. Within this option, a feature may help the user to select the right data in order to get the best efficiency (heliocentric inclination maximised) for the interplanetary transfer. In case of data inconsistency a warning box will be displayed (see annex).

Important: *those initial data are transformed with simple Kepler's laws wrt the focus in the radius and velocity wrt the focus for starting the integration of the trajectory. Hence those initial data do not include the effects of the perturbations neither the perturbing bodies effects: it is osculating Kepler orbital paramerters (computed at the current point radius and velocity). As consistency the reports also provide from radius and velocity wrt the focus the same osculating parameters using only simple Kepler's laws.*

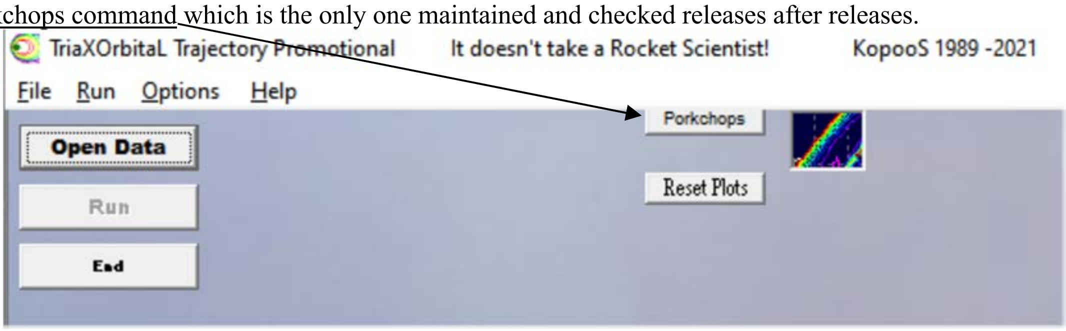
The principles of visualisation rely on the following parameters:

- The radius of reference: for viewing a reference circle with respect to the focus (GEO orbit or Mean Moon orbit for example)
- The radius max: this limit the output data to be shown (if not enough, please increase, save and re-run)
- The radius min: an interesting feature that allows showing only a **radius range** between min and max. This allows to show the very small evolution of the orbital data (setting this parameter to the GEO radius 20 km below allows to see the J2 or Sun-Moon perturbations,… and many other examples)

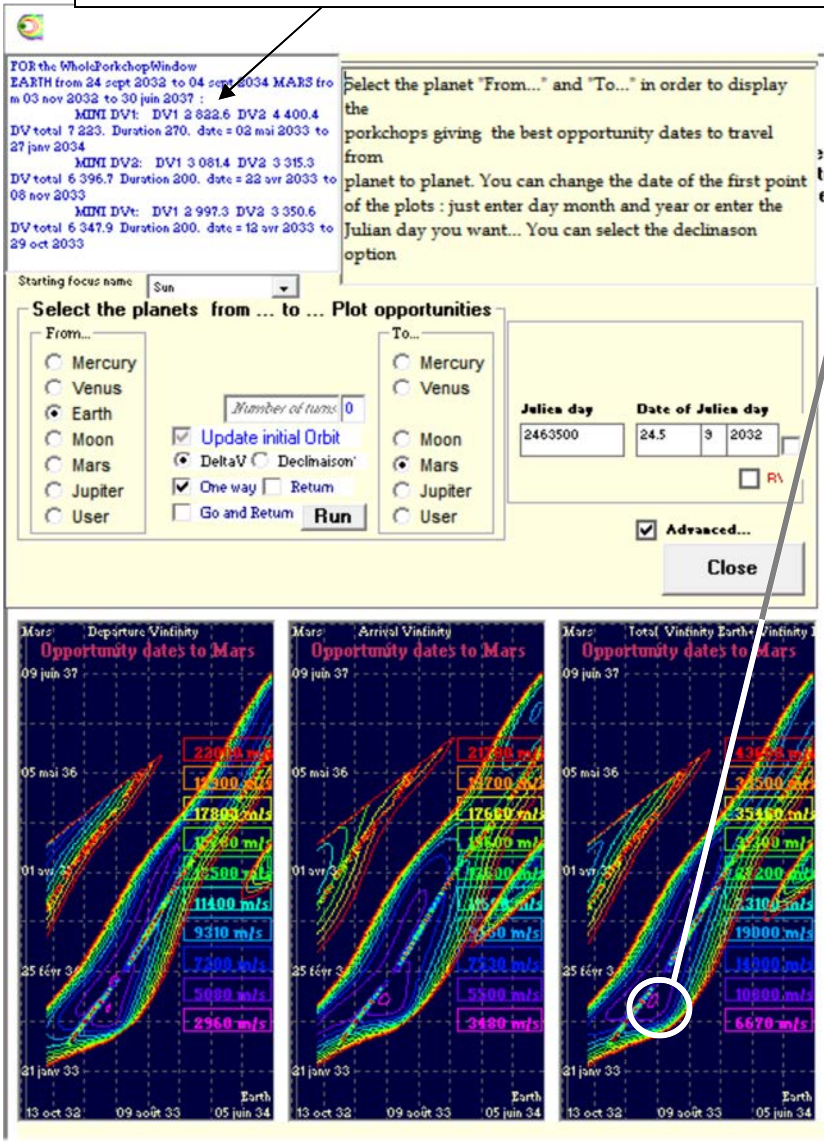
In order to help the user for repetitive tasks, a set of predefined options of "useful values" is also available. The user may adapt the data for its own use (and all can be saved in the sub-record).

When the checkbox Advanced… is checked, a set of advanced features allows the user to select:

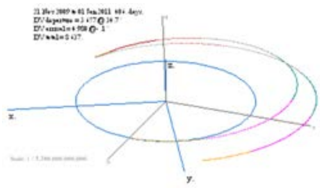
- An interesting Inclination zoom feature that is applied to the third coordinate, along Z axis (to be able to visualise the slight inclination changes due to the Sun-Moon perturbations for example)
- The drawing of computed orbit before the manoeuvre start
- The drawing of computed orbit after the last manoeuvre
- The date when the computation starts (particularly useful with the ephemerid of the planets or the Moon)
- Also when the checkbox Advanced… is checked and if the focus selected is the Sun or the Earth (or when they become selected), then an "opportunity dates from planet to planet" tool is provided. Sun should be selected as focus for heliocentric transfers, while if Moon is selected the focus changes to Earth automatically. Warning: when the user want to use this feature only, it is better to directly use the Porkchops command which is the only one maintained and checked releases after releases.



The partial Log file. The user shall double click in this box to view a full log file, etc.



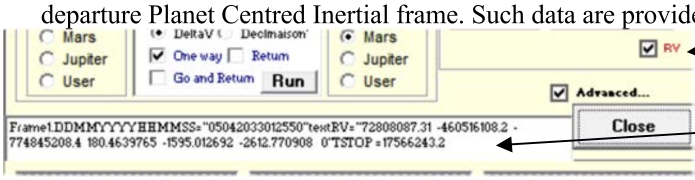
Once a start date and an arrival date are selected (with the mouse on the graph), then automatically, the changes of all the initial orbit parameters as well the Date are performed and the corresponding plot of the orbits and trajectory between the starting planet and the arrival planet are shown



In addition an Update feature has been added: this allows plotting easily transfer with accurate departure and arrival dates.

The tool displays the velocity to provide after the planet escape, abbreviated DV for Delta Velocity. DV is also called the infinity velocity (V_∞) of a planet centred hyperbolic branch. The declination of the DV can also be displayed when selecting the corresponding option and Run again the tool.
Of course those infinite or escape DV can be performed advantageously using the so-called Oberth effect, that is a DV performed near the perigee as low as possible above the planet surface.

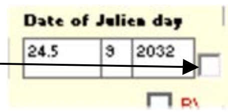
- It can be useful that the tool provides starting data in terms of Radius, Velocity (RV) of the orbit transfer in the departure Planet Centred Inertial frame. Such data are provided when selecting the checkbox RV.



Data: date, Rx,y,z,Vx,y,z , duration

- Contiguous porkchops

This feature is rarely used, but when needed, the user can click on the unnamed checkbox near the date text boxes.

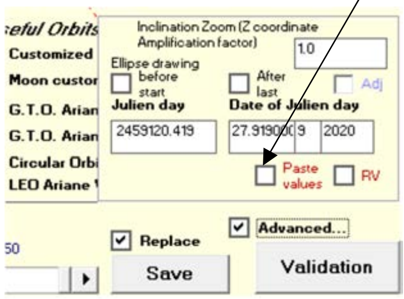


Automatically, the tool changes the starting date of the porkchop for the departure planet by the last date of departure from the porkchop previously computed and the tool run again the feature to get new porkchop. Once done, the unnamed checkbox becomes unchecked, ready for new use.

Note: For each such run a log file is filled with the best opportunities (minimum departure velocity "DV1" or minimum arrival velocity "DV2" or minimum total departure and arrival velocities "DVt"). See above "*The partial Log file* "

- Other particular features: not available in the current release

Data from previous runs: *if the user has copy paste results from previous run in the* text box area of the configuration window, *when selecting Advanced, a new checkbox "Paste values" allow the user to update the orbit data as well.*



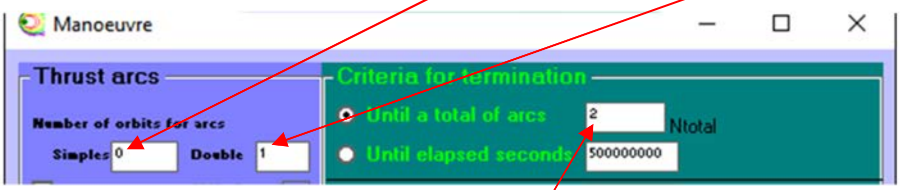
As before, one use the scrollbar to select the sub-record: its unique record number is displayed above the scrollbar . Then the user can modify the settings or change the sub-record title. Then the user can Save it with the new time stamp (or replace it if the checkbox Replace is checked) … or/and then press Validation.
Note if the checkbox Replace is not checked, the Save command generate a new unique sub-record number.

Window Orbit: tree of the available features

- **Scrollbar: records selection**
- **Checkboxes**
- **Hyperbola**
  - **Replace: records**
  - **Advanced…**
    - **Inclination zoom**
    - **Ellipse drawing before start**
    - **Ellipse drawing after start**
    - Special: Not available
    - **Paste values**
- **List box : Starting focus name : the reference frame is mentioned Equatorial or Ecliptic**
- **Save**
- **Validation*: command that allows to view the next window***

Window Manoeuvre, available with Open Data , command Validation...

In the program, a manoeuvre is defined by a number of orbits having thrust arcs as well as no thrust arc at all but with a long duration for the visualisation of the tiny perturbation coming from the Earth potential or/and from the other celestial bodies.
The apsides (ApoAstre and PeriAstre) arcs zone includes the settings of the manoeuvre. The criteria zone allows to set the criteria to be satisfied in order to terminate the trajectory and stop the computations.
It includes the feature to perform Number of orbits for arcs: one single (so-called Simples) or two thrust arcs (so-called Double) for "**repetitive thrust arcs strategies**":



- The total number of arcs = Simples + 2*Double + Last_Simples where the number of " Last_Simples" arcs deduced from the value set in Until a total of arcs N_{total} when such option is checked.

- The number of Simples arcs (one single thrust arc per orbit) is always performed with the thrusters set n°1.

- The number of Double arcs (first arc to be always performed with the thrusters set n°2, second arc with the thrusters set n°1 at a location centred in opposition - i.e. 180° after the first one- and so on). *After completion of that number of double arcs, the manoeuvre may continue the " Last_Simples" arcs on the basis of one arc per orbit with* always *the thrusters set n°2.*

  WARNING: the user shall check in the results that the thrusters sets n°1 and n°2 are used as wanted.

  ~~Note: The thrust arcs are performed until the number of arcs specified in the criteria "total number of arcs" is reached (even if that criteria is not checked). It recommended to write a large number for the total number of arc when selecting other criteria in order to not get the unexpected behaviour – the thrust stop before satisfying the criteria selected~~ -. Deleted because tool updated for avoiding such case.

- A check box "Altern. thrust pulses" for managing the particular case of thruster's sets used alternatively: set n°1 then set n°2 etc…

- A check box "Arcs specified in file" for specifying the arcs location by an external event file which is a simple text file starting by the time of the event thrust starts and followed by the time of the event thrust ends and so on.

- The amplitude of the half-arc of thrust
  - In degrees of true anomaly,
  - In minutes (of real time),
  - In degrees of eccentric anomaly.

- The "First thrust Arc Centered on" specify the location of the middle of the thrust arc (generally performed at the apogee or perigee): the user may select the apside option PeriAstre or ApoAstre or set the location directly in the Apside number text cell (*example 0 for perigee, 1 for apogee, 0.4 for an intermediate location at 180°*0.4= 72 ° of true anomaly between perigee and apogee, the rule being that the fractional part correspond to an angle that is the same fraction of 180° …*).

- The checkbox "no inertial" allows the user to let the shift of the apsidal line (due to mainly the J2 earth pole flatness perturbation or due to other bodies perturbation) to be taken into account for the next location of the thrusters pulse.

The criteria to stop the computation case are:

- Until a total number of arcs: when the count of individual arcs performed reaches that number, the program can stop the computations, terminate the display and finalise the viewing according to the relevant parameters.
  Note: as mentioned before, the value indicated here is always taken into account for performing the whole manoeuvre.

- A total duration (until elapsed number of seconds): when the elapsed time reach that number, the user is prompted for continuing the computations, and the program finalise the computations and viewing according to the relevant parameters.
  Note: Only when this criteria is selected, the number of points per orbit saved in the redraw graphic memory is adjusted in order that the whole trajectory corresponding to the total duration can be re-displayed (if possible, according to a total memory of about 96000 points) . For any other criteria, the points are saved in the redraw graphic memory at each Runge-Kutta step time until the redraw memory is full. This does not affect the current plot, but may restrict the redraw feature to that limited number of points.

- An orbital parameter criteria: Perigee altitude or Apogee altitude or eccentricity or $V_{infinity}$: when the orbital parameter reach that criteria when increasing up to or decreasing down to the criteria value, the program finalise the computations and viewing according to the relevant parameters. *Note that if the goal is to reach the minimum value of eccentricity, then the user shall set 0. in the cell corresponding to the* Eccentricity < *criteria.*

- Two particular combined criteria cases are provided:
  - the criteria Continuous orbit transfer can be used to compute continuous orbit transfers between GTO and a circular orbit that follows one of the Koppel patented strategies (for example according to the patent n° FR2 747 102).
  - the criteria Eccentricity then Apo. Alt. to perform first a manoeuvre in order to reach the criteria of eccentricity lower than and then to continue performing the manoeuvre until the Apo. Alt criteria is satisfied.

An advanced feature includes the complex perturbations from the other celestial bodies and the Earth particularities (J2 potential perturbations and Eclipses computations).

Perturbation due to bodies

- Perturbations are allowed from the bodies name to be selected in the scrolling list: Earth, Moon, Mercury, Venus, Mars, Jupiter, Saturn, Sun (*note the focus body is automatically remove from the displayed list*),

- Useful for interplanetary trajectories : the internal planets plus Mars with Sun, Mercury, Venus, Earth, Moon

- the external planets plus Venus with Sun, Venus, Earth, Moon, Mars, Jupiter,

- the bodies Sun, Mercury, Venus, Earth, Moon, Mars, Jupiter, Sat, *Uranus, Neptune, Pluto*.

The user can by hand type the orbital data of the selected body (the given time, Theta at the given time , RAAN, perigee argument, Inclination, Eccentricity, and the parameter Period day) to produce circular (elliptical setting the Eccentricity value to zero) frozen orbit around the focus. Circular restricted 3 body problem (**CR3BP**) can be simulated without any problem.

This feature of the complex perturbations may be used for the elementary studies for simplifying the complexity (changing step by step any orbital parameters of the body) as well as to amplify the effects. *This is exemplified when the Period day parameter (because that period day is used to compute the distance between focus and body) of the body is set to the tenth of its original value: the perturbation forces are very amplified and the consequences on the trajectory are visible without waiting or wasting a large amount of computation time.*

In the case of interplanetary trajectories, when the spacecraft approach one of the selected body, the corresponding plot of the trajectory with respect to that body (ecliptic frame) is automatically generated and displayed. As well, the part of the trajectory that may be perturbed by the body is displayed automatically in the main plot (ecliptic or geocentric equatorial frame).

The user can also rely on an internal ephemerid generator ([R 7] S. Bouiges formulae), that is valid for the current 21st century. *In addition the user can analyse the ephemerid given by that formulae when clicking on the command button ephemerid.*

Perturbation due to the Earth potential and Eclipses feature

If the focus is the Earth, the advanced zone allows to select the computation of the J2 (non spherical Earth gravitation potential due to the Earth polar oblateness) or/and the J2,2 (due to the Earth Equatorial ellipticity), a “x10” (ten times J2 and/or J2,2 to amplify the perturbations) and the management of the Earth shadow (Sun eclipses) suited for small solar electric propulsion satellites without batteries.

Perturbation due to the Sun Pressure

The Force due to the sun pressure, is computed with a pressure coefficient of 1.5 with a sun pressure of 4.63 E-6 N/m² at one AU (astronautic unit) acting on a frontal area given in the text box.
For other location than 1 AU, the sun pressure follows a law in 1/r².

Fixed inclination: **not yet available**

For some study cases, it is interesting to not consider the inclination changes. For such cases, a check box is available. At each full orbit, the inclination is set at its initial value.

Other features are not yet fully available: Shift of the opposition between thrust arcs, etc..

As before, one use the scrollbar to select the sub-record: its unique record number is displayed above the scrollbar . Then the user can modify the settings or change the sub-record title. Then the user can Save it with the new time stamp (or replace it if the checkbox Replace is checked) … or/and then press Validation.
This action will also close the three previous windows.
Note if the checkbox Replace is not checked, the save command generate a new unique sub-record number.

Window Manoeuvre: tree of the available features

- **Scrollbar: records selection**
- **Checkboxes**
  - **Altern. Thrust arcs are performed with thrusters set n° 1 then n°2 etc…**
  - **Degree True Ano.**
  - **Minutes of time**
  - **time minutes wrt apside: Half thrust arc in degrees are continuously computed to reach the specified duration set in minutes of time in the Half trust arc text box.**
  - **Degrees Excentric Ano.**
  - **No Sideral**
  - **Replace: records**
  - **Advanced**
    - **Manual**: Allows the user to perform pre-visualisation. Not fully available
    - **Complex perturbation**
    - **Data Bouiges : use the simplified ephemerid from [R 7] S. Bouiges formulae.**
    - **Command Ephemerid: to generate a file of ephemerid**
    - **J two : available with Earth focus only**
    - **Equatorial ellipticity: available with Earth focus only**
    - **x10 : available with Earth focus only**
    - **Inclin : Force the inclination to be removed from one orbit to the other**
    - **Eclipses : available with Earth focus only: thrust is turned off in eclipses**
    - **Sun Pr.: compute the force due to the sun pressure on the solar arrays**
    - **Aero: compute the drag force due to the atmospheric density**
  - **Lagrange**: (*only available when MOON is selected in the first perturbing body*) **adjust by performing a search loop (based on the initial position of the Moon given by its time at 0) of trajectory until the distance of one its points is exactly E-M L2. Once a solution is found, plots are cleaned and a delta V is added at L2 in order to follow the trajectory at the unstable libration point L2**
    - **L1 not L2: Sub-specification of Lagrange Adjust : by default the libration point selected is L2. For specifying L1, this checkbox must be selected.**
- **Save**
- **Plus: allow to smoothly change the colours with the time (useful for long duration cases)**
- **Validation**: ***command that allows to view the next window***

Note1 : when clicking on the checkbox **Sun Pr.** a textbox is displayed for the input of the area (in m²) exposed to the sun

Note 2: when clicking on the checkbox **Aero** a textbox is displayed for the input of the CdS (in m²) the area exposed to the drag times its drag coefficient.

The type of atmospheric model has by default the code 1 meaning a Moderateactivity.

Atmospheric model code according to "ECSS-E-ST-10-04C Space environment":
0 for Lowactivity
1 for Moderateactivity
2 for Highactivity(longterm)
3 for Highactivity(shortterm)

Other type of atmospheric model can be selected by writing a code after the CdS value and a space.

Window Database record selected , last window available with Open Data

The window displays the sub-records information and let the user add comments for traceability like the record title and a user defined dataset number (*which may include fractional part*).

The user can Save the record as a new record (when selecting the corresponding option), or replace the previous one, or replace a specified record.

If the sub-directory "/Data" is read-only protected, the Save command abort, and a warning is added to the title.

Click on Validation to allow the further processing.

Because one of the main rules for the use of the program is the traceability of each computation, if the user did not perform a Save of his database record when clicking on Validation, a cool warning message is displayed in that case.

Window Database record selected: tree of the available features

Options

- **Look at other dataset... : allow to continue the Setting of the Data again.**
- **Terminate the Data : allow to continue for the computations.**

Save

- **Options**
  - **Replace the previous.**
  - **New**
  - **Replace the n°**
  - **Do not save**

Validation*: command that allows closing all the SetData windows.*

The next command is generally the Run: command to compute the orbit manoeuvre. It is also possible to come back the previous command Open Data for any change required.

Windows & commands displayed after the command Run

During the computations, the user can use the buttons from the main window TriaXOrbitaL Trajectory in order to visualise in live the summary results, of visualise the log of warnings and events.

The main window is reduced in size, with the following button commands added:

- Pause (Continue)
- Visual. Results
- Visual. Log

In the Run command context, the logically no more useful command like Open Data or Constellation are no more enabled.
The user can perform a pause in the computation at any time when clicking on the Pause button. The button change to a new button labelled Continue in order to allow the restart of the computation.

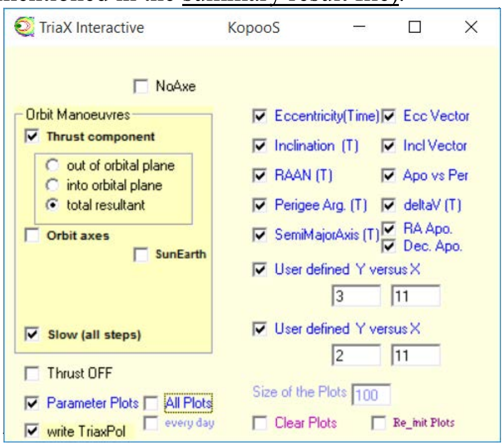
The Visual Results button allows the user to visualise in live a part of the file Results_Tmp.xls that is being generated.

Refer to the chapter Graphic Viewer for the use of the further commands available.

During the process the program may produce some minor warnings or events logs that are recorded in a specific text zone called *log textbox*. Each time a new entry is added to that zone, the background colour is changed. The major warnings are displayed in specific message boxes that stop the process until the user has selected one command. The content of the all warning messages is saved temporarily in file “Warning_Tmp.xls”

A context adapted tool box named TriaX Interactive is displayed. It allows the user to select some particular features of the plots, as indicated.

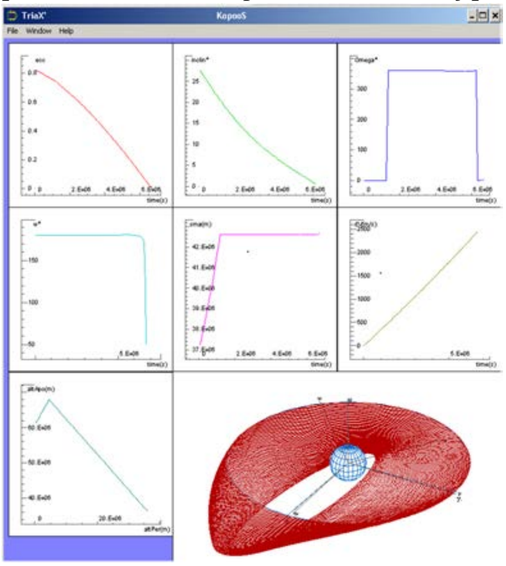
A particularly interesting feature is the Parameter Plots check box that allows to select one or all of a predefined set of 2-dimensional dynamic plots, an example of output is shown here under (eccentricity, inclination, semi-major axis, … versus time, Eccentricity vector, Inclination Vector, Apogee Right ascension, Apogee declination and some used defined plots with respect to the column numbering mentioned in the summary result file).



Note 1: the plots are performed at every events --every orbit, and/or before and after each thrust arcs--. The user can perform the zoom of that plot as usually (right mouse button down to select a part to zoom).

Sub Note 1 : this feature is particularly interesting when there are numerous orbits.

*Note 2: In addition features “**Re-init plots**” allows for starting new plots from the current date of the computations; “**Clear plots**” to not show the plots; and the **size of plots** is customisable.*



Again in the context tool box TriaX Interactive, the user can select the Rotating earth checkbox in order to describe the relative trajectory of the satellite for a ground station from Earth.

Also an In-orbital plane only checkbox can be selected to freeze the plot in the current orbital plane and to get thus only the apogee-perigee axis evolution and rotation in that plane. This is automatically set when Earth J2 perturbation is used.

For some case, the files saved on the hard disk may become huge in size, and their writting slow down the runs. In such case, the user can uncheck the checkbox 

in order to suspend the writing on the files that record all the computation's steps (TriaxPol.xls; TriaxCarT.dat).

Windows & commands displayed with the command Constellation

This command has been build in order to view some of the constellation orbits, and their peculiarities. It is available under the menu "Options".

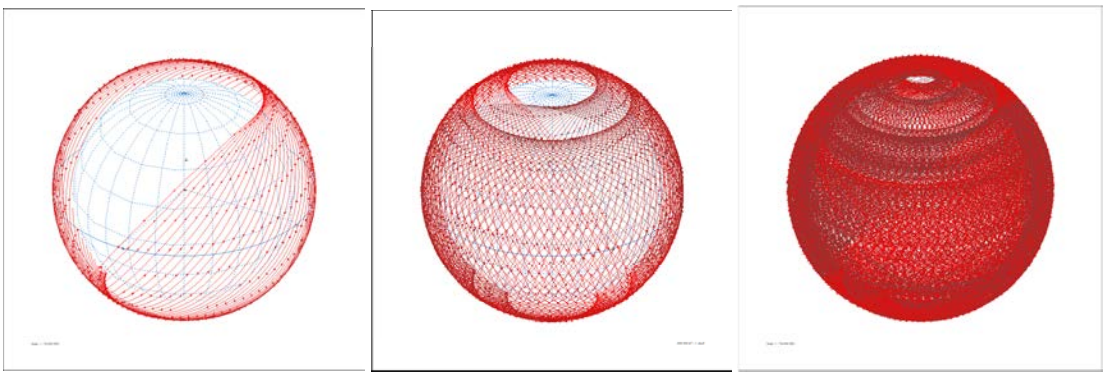
The mouse allows the automatic display of a list of options of predefined constellations when moving above the button:

- GPS (24 satellites),
- Iridium (66 satellites),
- Globalstar (48 satellites),
- Teledesic 1° (840 satellites),
- Teledesic 2° (264 satellites),
- Skybridge (32 satellites),
- Galileo (27 satellites)
- StarLink* (42000 satellites, in 15 phases)
- OneWeb* (6400 satellites)
- Kuiper* (3300 satellites)
- GuoWang* (13000 satellites)

  * Note for large constellation, settings are performed according to the corresponding .txt file in the data sub-directory

A last option "User defined" let you define precisely any other one, including the design J. Walker TPF.

Select one, double click to launch the run (or move the mouse on the right and click the command **Constellation**).



Example of StarLink constellation from phase 1 to full deployment of the second generation

All the constellation trajectory data and satellites numbering are saved in the file "TriaXOrbitaL\xls_doc**TriaxCar.txv**" and if the tool http://www.kopoos.com/FTP2/TriaXExcelPro.zip is already installed, the constellation is shown in the nice interactive 3D windows of "TriaXExcelPro"…

Click on the option "user defined" in order to set the display of your own constellation. This is performed in the window **Constellations user settings**. The parameters names in that window are meaningful without any further need of explanation. Click on the command **OK** to display the constellation.

Double-click on the plot to change the orientation,… Refer to the chapter Graphic Viewer for the use of the further commands available.

* Note for large constellation, every phase is displayed for information.

Windows & commands displayed after the command Sun System

This command has been build in order to compute the integration of the sun system and to show its evolution (problem with n corpus). In order to not overload the user manual, this feature has been deactivated.

The End command: to Exit the programme.

The program termination occurs when the user click at any time on the command End. This command close all the windows related to the programme.

Note: If some windows of the programme are closed individually, the program core may still be present in the computer memory. In that event, to terminate all the activities of this program close it in the taskbar.

It is highly recommended to end the program (and eventually save some of the results files by renaming them) before starting a new trajectory computation.

8. Graphic Viewer

This chapter describes the features enabled during the trajectory computations.

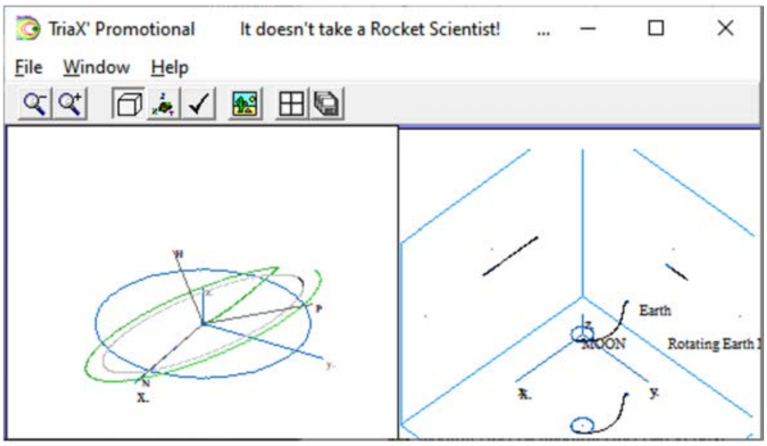
A window so-called View allows to set orientation of the displayed view with respect to the centre of the world (that is the focus).

Simply, use the mouse and click on the X axis to choose the right orientation. A database can also be managed, using as before the **scrollbar**, **replace** checkbox and the **save** command.

- The advanced feature (select the checkbox "Advanced" or simply move the mouse in that area) of the window View, allows selecting the sometime very useful cubic view (a 3D plot together with the orthogonal projections in the 3 main planes; like if the 3D plot were put into a cube) by checking the checkbox "**Cubic view**".
- In addition a text zone allows the user to select a **cubic view size** of the plot: 0.5 a half size, 2.0 for a double size.
- A second text zone labelled **Size%** allows to change the size in percents of the nominal plot size. *Note: In the two cases, it is not recommended to set a very large size because the graphic memory may be overloaded and the programme may **abort**.*
- By checking the checkbox "**Skip this window at launch**" and saving (or replacing) as the record n°1, the next time the user will not be prompted by that window. *The plot will use by default the settings of the record n°1. The user can anyway change the orientation by double clicking on every plot.*
- **Print the plot** can be used to perform direct plots on the current printer (or on PDF printer if any).
- **Movies** feature performs images recordings of regularly spaced images of a dynamic plot. This feature is useful when the computations have started (see below).

Use the Validation button to continue.

The next window that is displayed contains all the plots generated by the program. That window is TriaX'.



Several basic icons have been added:

Reduce size, increase size of the last selected plot

Cubic view (see above)

A panorama of Iso views while clicking successively on this icon

Several right plots (with scale)

Copy a plot ready to paste into a doc

Tile and cascade of plots (when there are several)

The user can change the **orientation** in live (during the computations) when **double-clicking** the mouse left button on a plot.

Movies feature performs images recordings of regularly spaced images of a dynamic plot. This feature is useful when the computations have started: on the base of the current status of the computations, the program compute the events for saving regularly the images of the plot. The images are numbered regularly from the first point plotted to the last one. The number of images (20 by default) can be changed in the corresponding textbox *–note: each image is saved in bmp format, so a large number of images may request a large free space on the hard disk–*. The location where the image shall be saved can also be changer by the user in the corresponding textbox. With other software, like Paint Shop Pro® Animation, one can use those images to create gif animated image which produce a movie of the dynamic plot.

The user can perform in live any **zoom** from any plot when selecting a zone with the **right button** of the mouse down : an other specific plot window is displayed for that feature.

Move of the plots are performed with the mouse right button down.

The user can change the background **colour** when double-clicking the right button on each plot.

The user can change the **size** of the plot when the mouse is placed near the boundary of the plot and keeping the right mouse button down.

Double-click into the window (not into a plot) allows to automatically change the view to a **cascade** of plots or **tiles** of plots alternatively.

Etc…

9. Glossary and abbreviations

Term	***Description***
Altitude	Distance between the planet surface and the spacecraft. *Easier to handle than the radial distance that start at the planet centre.*
ApoAstre	Idem Apogee for any other planet than Earth.
Apogee	The orbital location that is at the highest distance from the Earth. Apply to the satellites orbiting around the Earth.
AU	Astronautic unit=149 597 870 000m The mean distance between Earth and Sun.
Button command	Software feature: 
Checkbox	Software feature: one or many possibilities can be selected together. 
Cubic view	A 3 D plot together with the three main plane projections.
Eccentric anomaly	An orbital characteristic: the angle based on the ellipse centre, between the perigee line and the current location on the ellipse (quantity between 0° and 360°).
Eccentricity	An orbital characteristic Eccentricity *e* determines the shape of the orbit. It is a unit less geometric constant with a value between 0 and + infinite. A pure circular orbit has an eccentricity of zero.
ECI frame	Abbreviation “Earth centred inertial” frame. A Galilean frame that may be equatorial or ecliptic.
Declination	An orbital characteristic related to the direction of a vector: angle with respect to a reference plane: when considering hyperbolic branches for hyperbola centred on the Earth, the reference plane is the Equatorial plane; for other planet, the Ecliptic plane can also be taken as reference (quantity between -90° and +90°). For a full north vertical hyperbola branch, declination is +90°.
DV or ΔV	Delta Velocity. This is the value of the velocity to be added to some orbital velocity for performing an orbit transfer. The addition is a vector addition in 3 dimensions unless the vectors are collinear. It is also the ideal velocity of the Tsiolkovski equation (see below)
Ecliptic	The ecliptic is the orbital plane of the Earth around the sun.
Ecliptic frame	A Galilean frame for which x is the direction of the vernal point, **z** the north side orthogonal to the ecliptic plane.
Equatorial frame	A Galilean frame for which x is the direction of the vernal point, **z** the north side orthogonal to the Earth equatorial plane.
E- M L 2	Lagrangian point L2 of the system Earth Moon
Focus	The main celestial body that attract a spacecraft (at least when the computation starts).
Galilean frame	A frame having fixed direction relative to the stars, and that is not accelerated...
GEO	Abbreviation “Geostationary Earth Orbit” It is a unique circular orbit lying in the Earth equatorial plane having an orbital period equal to the one of the Earth (23h56’4.0905’’…). Depending on the Earth model taken into account, the corresponding altitude (w.r.t the Earth equator) may vary: from 35 786 034.7 m for a perfect spherical Earth to 35 786 560 m for an oblate Earth… Other perturbing bodies (Moon, Sun) also change that altitude. *Note those altitudes do not take into account the tiny effects due to the Earth rotation period changes that come from the Earth axis precession and neither coming from the mean solar day slowing caused by the moon tidal effects. Reference: R.A. Nelson.* Note 50% of the references define the GEO as the GEOsynchronous earth orbit or Geosynchronous Earth Orbit
Geostationary	A point in the sky that stays fix when viewed form the Earth surface. *Please note that some reference call that feature Geosynchronous.*
Geosynchronous	A point in the sky that have simply the same period of rotation as the one of the Earth. *Please note that some reference call that feature Geostationary.*

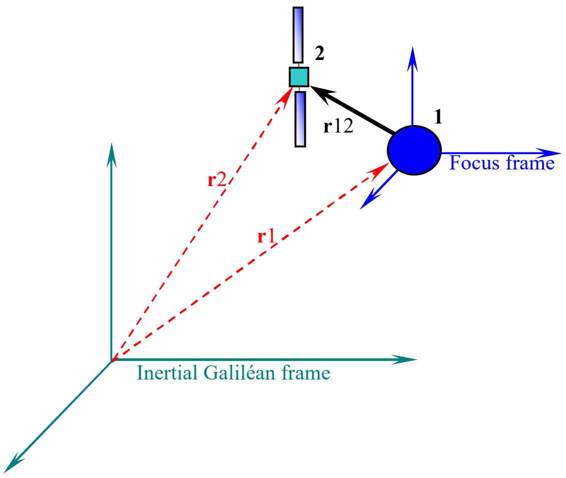
Term	*Description*
GSO	Abbreviation "Geosynchronous Earth Orbit" Note 50% of the references define the GEO is as the Geostationary earth Orbit
GTO	Abbreviation "Geostationary Transfer Orbit"
Inclination	An orbital characteristic (quantity between 0° and 180°).
Inertial	See sidereal.
Isp	Specific impulse: a performance parameter for the thrusters that is defined as the impulse produced (F. dt) divided by the mass exhausted (dm). In the program, the conventional definition of Isp is taken into account: Isp= F /(g0.q) with q= dm/dt and g0 = 9.80665 m/s², F in N, q in kg/s.
J2	A gravitation potential characteristic: for non spherical gravitational bodies (like the Earth), the potential may be described by a sum of harmonic series with non-dimensional coefficients. For the Earth, the coefficients J2 (due to the Earth oblateness) may produce the most important orbital effects, J2=0.001083. For some particular inclined orbits, the effect of the J2 can be annihilated.
J2,2	For the Earth, the coefficients J2,2 (due to the Earth equator ellipticity) may produce the most important orbital effects after the previously seen J2, J2,2=1.803E^{-6} . For some particular location in the equatorial geostationary orbit, the effect of the J2,2 can be annihilated.
LEO	Abbreviation "Low Earth Orbit"
Line of node	An orbital characteristic: the line formed by the intersection of the orbital plane and the reference plane "xOy" (i.e : the equatorial plane or the ecliptic plane). The ascending node is defined at the location where the satellite pass from the South to the North, the North being the direction Oz in the reference frame
LLO	Abbreviation "Low Lunar Orbit"
LMO	Abbreviation "Low Mars Orbit"
Longitude	An orbital characteristic (quantity between 0° and 360°). Typically, a longitude is an angle in the reference plane. However some longitudes are coming from composite planes. The rotating Earth longitude is the angle in the equatorial plane from the Greenwich meridian.
Option	Software feature : only one of the possibility of the same context can be selected at one time 
Orbit Parameters	In order to define the trajectory of a satellite in space, orbital parameters are required. ○ The shape of an orbit is described by two parameters: the semi-major axis **a** or **sma** and the eccentricity **e**. ○ The position of the orbital plane in space is specified by means of another two parameters: the inclination **i**° and a longitude of the ascending node **Ω**°, see Right Ascension of the Ascending Node (RAAN). ○ The orbit orientation in the orbital plane is defined by the argument of perigee **ω**° (it is always supposed that the attracting body is located at one focus of the ellipse). ○ The position of the satellite in the orbital plane is defined by the true anomaly **φ**°..
PeriAstre	Idem Perigee for any focus other than Earth or Sun.
Perigee	The orbital location that is at the lowest distance from the Earth.
Perihelia	Idem Perigee for any spacecraft or planet orbiting around the Sun.
Perigee argument	An orbital characteristic: the argument of perigee **ω**° determines the rotation of the orbit in its own plane. This parameter is the angle of the perigee w.r.t. the ascending node, following the satellite rotation. The argument of perigee is undefined in the case of circular orbits, since the perigee is undefined
Period	An orbital characteristic: the duration the spacecraft takes to perform a rotation of 360° around its focus. Depending on the angle taken into account, the period may be sideral, tropic, anomalistic etc… In the TriaXOrbitaL program, the period is computed from the Kepler formulae, $Period = 2*\pi\sqrt{\frac{sma^3}{G*m_{focus}}}$ thus when the computation take into account some perturbations (like J2 or Sun/Moon perturbation) the value reported by the Kepler

Term	Description
	formulae do not reflect accurately the real value of the period.
Period of a perturbing body	For the Moon orbit in the case of Circular restricted 3 body problem (**CR3BP**), the Moon period formulae is $Period = 2*\pi\sqrt{\frac{sma^3}{G*(m_{Earth}+m_{Moon})}}$ This provides as well the equation for the Moon semi major axis (sma) wrt ECI when the Moon period is given as it is the case when in "Window Manoeuvre " the Bouiges check box is not selected. This formulation is also used for any other perturbing body described by a circular orbit in the setting: $Period = 2*\pi\sqrt{\frac{sma^3}{G*(m_{Focus}+m_{Perturbing\_body})}}$.
Polar frame	Local frame based on the radius from focus unit vector, the unit vector normal to the radius in the orbital plane and the unit vector normal to the orbital plane.
RAAN	Abbreviation “Right ascension of the ascending node”
RK	Abbreviation “Runge-Kutta”: a clever kind of integration based on the Taylor formulae. The order of RK is in important characteristic: higher the order, lower the integration error. However, the number of times the function is computed increases rapidly with the order. So that the method order should be optimised.
Scrollbar	Software feature : 
Secular	An orbital characteristic (or the mean trend of it) that vary with the time without short or medium periodicity. Most of the time, the secular perturbations shall be corrected with appropriate propulsion devices.
semi-major axis semi-minor axis sma smia	The semi major axis abbreviated “a” or “sma”of an ellipse is defined by the parametric equation of the ellipse x=a.cos (u) and y= b.sin(u), with “b” or “smia” the semi-minor axis, and u the eccentric anomaly
Specific impulse	A quantity that shows the performance of a thrusters: higher the Specific impulse, higher the performance. See Isp.
Sidereal	With respect to the stars. Synonym to inertial.
Textbox	Software feature: object in a window that display text with eventually vertical or horizontal scrollbars
Thrust	The force produced by thrusters.
Thruster	A device that produce a force. The force, generated in the opposite direction of the exhausted particles or gas, is called the thrust.
Thruster set	A number of thrusters used simultaneously.
True anomaly	An orbital characteristic: the angle based on the ellipse focus (the focus where the celestial body is), between the perigee radius and the current radius (quantity between 0° and 360°).
Tsiolkovski equation	It gives the ideal velocity provided by a propulsion system $\Delta V = g_o Isp.Ln\left(\frac{M_i}{M_f}\right)$ where M_i is the initial mass of the rocket, M_f the final mass of the rocket after having run the rocket engine characterised by its Isp (see above)
Vernal	The point in the sky given on the first day of Spring when from Earth looking at the Sun. (the sun being in the pisces constellation in Spring, the Vernal point also is there). The point is abbreviated as γ. The advantage of this point is that the line from Earth to Sun and to γ is common to the Equatorial Inertial Frame and to the Ecliptic Inertial Frame on the first day of spring. In the software TriaXOrbital, the axe X points toward γ, the Vernal point. This point is also called as first point of Aries…
Window	Software feature: object on the screen that has a title bar and other features.
w.r.t.	Abbreviation meaning “with respect to”

Annex 1: Equations and Mathematics elements

Galilean frame and relative frames: solution of the 2 bodies problem

Earth at the focus --1-- , and a satellite --2--



$\vec{F} = m\vec{\gamma}$ is true for a galilean time reference frame

Hence the absolute satellite acceleration is

$$\vec{\gamma} = \vec{\gamma}_2 = \frac{d^2\vec{r}_2}{dt^2},$$

Here $\vec{F}$ is the force provided by the focus 1 on the spacecraft 2

$$\vec{F} = \vec{F}_{12} = -G\frac{m_1.m_2}{r_{12}^3}\vec{r}_{12} \quad \text{this gives : } \frac{d^2\vec{r}_2}{dt^2} = -G\frac{m_1}{r_{12}^3}\vec{r}_{12}\ .$$

Similarly, the force provided by the spacecraft 2 on the focus 1, $\dfrac{d^2\vec{r}_1}{dt^2} = -G\dfrac{m_2}{r_{21}^3}\vec{r}_{21}$

We want write a similar equation $\vec{F} = m\vec{\gamma}$ but in the focus reference frame,

with the acceleration, $\vec{\gamma} = \dfrac{d^2\vec{r}_{12}}{dt^2}$ and $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$$\frac{d^2\vec{r}_{12}}{dt^2} = \frac{d^2\vec{r}_2}{dt^2} - \frac{d^2\vec{r}_1}{dt^2} = -G\frac{m_1}{r_{12}^3}\vec{r}_{12} + G\frac{m_2}{r_{21}^3}\vec{r}_{21}$$

$$\frac{d^2\vec{\mathbf{r}}_{12}}{dt^2} = -\mathbf{G}\frac{(\mathbf{m}_2 + \mathbf{m}_1)}{\mathbf{r}_{12}^3}\vec{\mathbf{r}}_{12}$$

Perturbation forces due to the other bodies (Moon, Sun, ...)

As previously, the radius vector from the focus (Earth) to Satellite is : $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$

$$\frac{d^2\vec{r}_{12}}{dt^2} = -G\frac{(m_2 + m_1)}{r_{12}^3}\vec{r}_{12} - G\frac{m_{moon}}{r_{moon\_2}^3}\vec{r}_{moon\_2} - G\frac{m_{moon}}{r_{1\_moon}^3}\vec{r}_{1\_moon} - etc...$$

central term always managed — moon perturbation when required — quasi constant term

The formulation even if called Perturbation represents actually the complete equations for computing a spacecraft trajectory, with or without thrust events, in the solar system.

Perturbation forces due to the non spherical earth potential

The Earth's external gravitational potential $V(r,\varphi,\lambda)$ for $r > R_e$ can be developed into a series of zonal spherical harmonics and sectorial & tesseral spherical harmonics, forces are given by $\frac{F}{M} = -\nabla V$. *Note : in 50% of the references, one speak of force function U, that is the opposite of the potential $U = -V$, and thus forces are* $+\nabla U$. The Earth's external gravitational potential V vary from zero at an infinite location to a negative value at the Earth surface.

$$V(r,\lambda,\varphi) = -\frac{G\cdot m_e}{r}\left[1-\sum_{n=1}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_n(\sin(\varphi)) + \sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n P_{n,m}(\sin(\varphi))\{C_{n,m}\cos(m\lambda)+S_{n,m}\sin(m\lambda)\}\right]$$

or more commonly, with “n” starting at 2 because the terms for n=1 are all null for the earth, and $J_{n,m}$ negative:

$$V(r,\lambda,\varphi) = -\frac{G\cdot m_e}{r}\left[1-\sum_{n=2}^{\infty}\left(\frac{R_e}{r}\right)^n J_n P_n(\sin(\varphi)) - \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{R_e}{r}\right)^n J_{n,m} P_{n,m}(\sin(\varphi))\cos(m(\lambda-\lambda_{n,m}))\right]$$

with:

r : *the distance to the Earth centre of mass,*

φ : *the latitude from the equator toward the North,* $[-90,+90°]$

λ *the East longitude with respect to the Greenwich meridian*

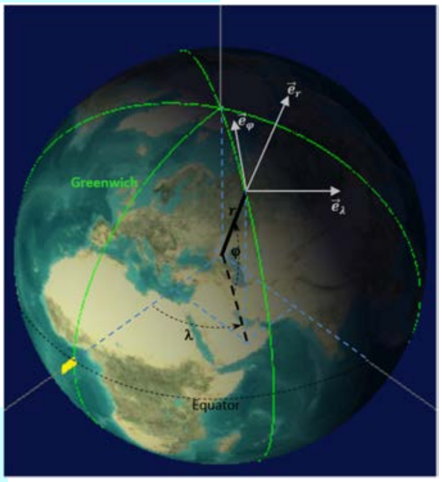
R_e *the equatorial radius of the Earth*

$G.m_e$ *the Earth gravitation constant*

J_n *the zonal non dimensional terms* $= -C_{n,0}$

$J_{n,m}$ *the sectorial & tesseral terms (negative values* $J_{n,m} = -\sqrt{C_{n,m}^2 + S_{n,m}^2}$ *)*

$\lambda_{n,m}$ *the sect. East long. with respect to Greenwich,* $\lambda_{n,m} = \tan^{-1}(S_{n,m}/C_{n,m})$



and the Legendre polynomials:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}$$

$$P_{n,m}(x) = (1-x^2)^{\frac{m}{2}}.\frac{d^m P_n(x)}{dx^m}$$

$$P_2(\sin(\varphi)) = \frac{1}{2}.(3.\sin^2(\varphi)-1)$$

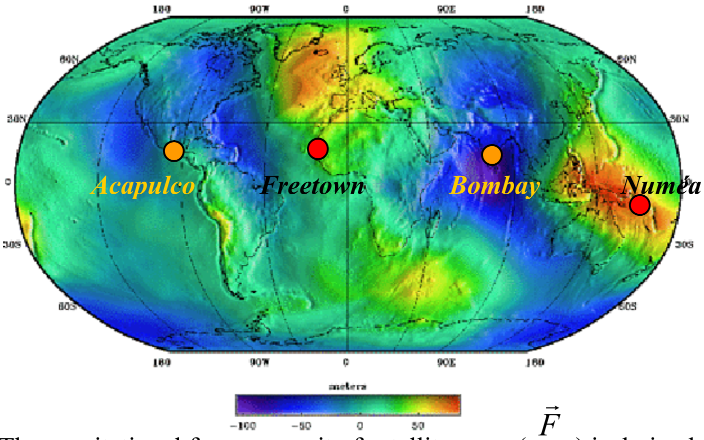
$$P_{2,1}(\sin(\varphi)) = 3.\sin(\varphi).\cos(\varphi)$$

$$P_{2,2}(\sin(\varphi)) = 3.\cos^2(\varphi)$$

$$P_3(\sin(\varphi)) = \frac{5}{2}.\left[\sin^3(\varphi) - \frac{3}{5}.\sin(\varphi)\right]$$

Because $\varphi \in [-90,+90°]$, $\cos(\varphi)$ *is always >=0, so* $\sqrt{1-\sin^2(\varphi)} = \cos(\varphi)$

Note that $\lambda_{2,2}$ *is near the city of Freetown, and by quadrature, the other linked cities are Bombay (stable), Numéa and Acapulco (stable), see figure (background courtesy of TBD)*



The gravitational force per unit of satellite mass ($\frac{\vec{F}}{M}$) is derived from the potential by the well-known equation:

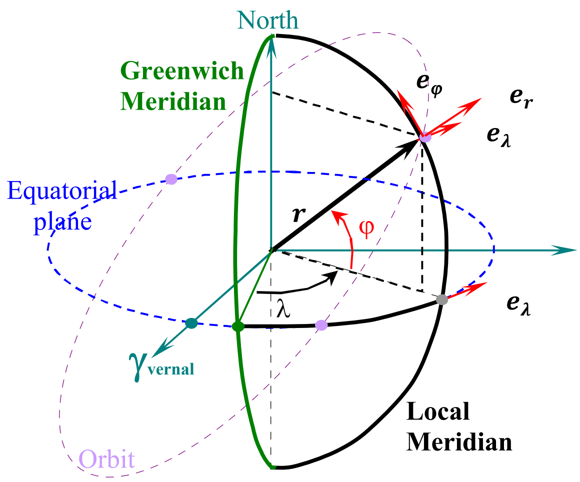
$$\frac{\vec{F}}{M} = -\vec{\nabla}V$$

That is in Equatorial/North spherical coordinates with respect to the rotating Earth.

$$\frac{\vec{F}}{M}(r,\lambda,\varphi) = -\vec{\nabla}V(r,\lambda,\varphi)$$

Thus, in the same Equatorial/North spherical coordinates frame (direct frame based on $\boldsymbol{e_r}$, $\boldsymbol{e_\lambda}$, $\boldsymbol{e_\varphi}$),

$$\frac{\vec{F}}{M}(r,\ \lambda,\ \varphi) = -\left| \begin{array}{c} \dfrac{\partial V(r,\ \lambda,\ \varphi)}{\partial r} \\ \dfrac{\partial V(r,\ \lambda,\ \varphi)}{r\cos(\varphi)\partial L_G} \\ \dfrac{\partial V(r,\ \lambda,\ \varphi)}{r\partial\varphi} \end{array} \right.$$



When only J_2 is considered:

$$\frac{\vec{F}}{M}(r,\ \lambda,\ \varphi) = +\frac{3}{2}J_2 GM_e\frac{Re^2}{r^4}.\left| \begin{array}{c} 3.\sin^2(\varphi)-1 \\ 0 \\ -\sin(2\varphi) \end{array} \right.$$

When only $J_{2,2}$ is considered *(note $J_{2,2}$ is a negative value)*:
:

$$\frac{\vec{F}}{M}(r,\ \lambda,\ \varphi) = +J_{2,2} GM_e\frac{Re^2}{r^4}.\left| \begin{array}{c} 9.\cos^2(\varphi).\cos(2.(\lambda-\lambda_{2,2})) \\ 6.\cos(\varphi).\sin(2.(\lambda-\lambda_{2,2})) \\ 3.\sin(2\varphi).\cos(2.(\lambda-\lambda_{2,2})) \end{array} \right.$$

When only J2 and $J_{2,2}$ are considered the force, per unit of satellite mass, is the superposition of the two previous cases (simple addition).

Other equations used

- Sphere of influence : the definition of the radius of the sphere is given by example for the Moon:

$$R_{sphereMoon} = sma_{moon}.\left(\frac{1}{2^{0.5}}.\frac{G.m_{moon}}{G.m_{earth}}\right)^{2/5}$$

with sma= semi-major axis of the Moon orbit around the Earth given by: $OrbitalPeriod_{Moon} = 2.\pi.\sqrt{\frac{sma^3_{moon}}{G.m_{earth}}}$

- Greenwich right ascension at midnight in degrees = (100°.152475 + 0°.7694 . cc + 0°.0003871 . cc² + 360° * (100 * cc - Int(100 * cc))), with JD for Julian day, Mean Julian day : MJD = Int(JD - 2400000.5) , cc = (MJD - 36934.5) / 36525, according to *reference* [R 4] *Chobotov, (and checked with respect to other references).*

Program constants

Earth rotation speed = 360 / 86164.0905 °/s (used for the Earth rotating frame, J_2 and $J_{2,2}$ computations)

R_{Sun} = 695 000 000 m	$G.m_{Sun}$ = 1.32715E+20 m³/s²	UMA= 149 597 870 000 m
$R_{Mercury}$ = 2 439 700 m	$G.m_{Mercury}$ = 22 031 100 060 000 m³/s²	Earth gravitation harmonic terms :
R_{Venus} = 6 050 000 m	$G.m_{Venus}$ = 325 000 000 000 000 m³/s²	J_2=+0.0010827 *(no dimension),*
R_{Earth} = 6 378 137 m	$G.m_{Earth}$ = 398 600 500 000 000 m³/s²	$J_{2,2}$=-1.803E-6 *(no dimension),*
R_{Moon} = 1 738 000 m	$G.m_{Moon}$ = 4 902 000 000 000 m³/s²	$J_{3,1}$=-2.195E-6 *(no dimension),*
R_{Mars} = 3 400 000 m	$G.m_{Mars}$ = 42 900 000 000 000 m³/s²	$J_{3,3}$=-0.221E-6 *(no dimension),*
$R_{Jupiter}$ = 71 492 000 m	$G.m_{Jupiter}$ = 1.26700E+17 m³/s²	$\lambda_{2,2}$=-14.927° east,
R_{Saturn} = 60 268 000 m	$G.m_{Saturn}$ = 3.79391E+16 m³/s²	$\lambda_{3,1}$=7.06° east , $\lambda_{3,3}$=21.20° east.

Note R_{Sun} and R_{Earth} are used for the Sun Eclipses computations.

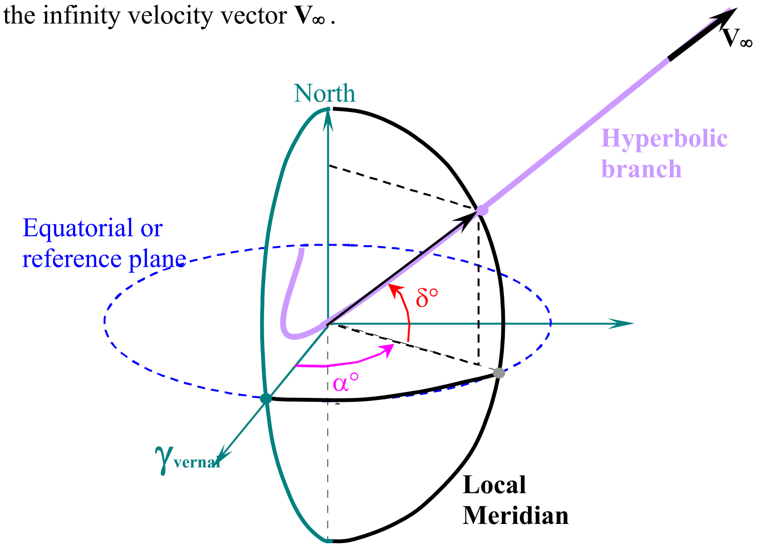
Main orbital characteristics

Only six parameters are needed for the definition of an elliptic orbit wrt the focus.
The orbital parameters are: sma, e, i, Ω, ω, $\varphi(t)$. Generally, with no perturbing force, only the angle « true anomaly » φ, is a function of the time.
We can transform easily those 6 parameters into the 6 Cartesian co-ordinates of position and velocity ($\mathbf{r}(t)$, $\mathbf{v}(t)$) and vice versa using the simple Kepler's laws with respect to the focus.

Hyperbola

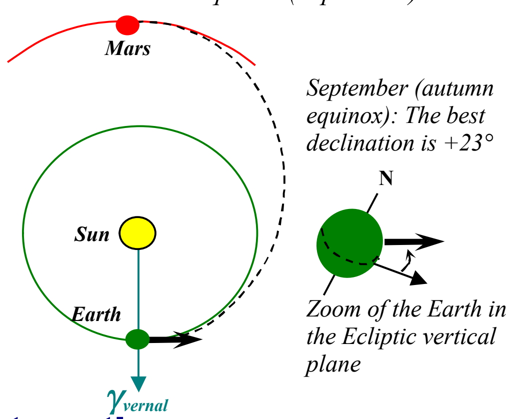
Only six parameters are needed for the definition of a hyperbolic orbit wrt the focus.
The orbital parameters are: $\mathbf{V}_\infty$, e, i, Ω, ω, $\varphi(t)$. V_∞ is the infinity velocity on the hyperbolic branch. Generally, with no perturbing force, only the angle « true anomaly » φ, is a function of the time.
We can transform easily those 6 parameters into the 6 Cartesian co-ordinates of position and velocity ($\mathbf{r}(t)$, $\mathbf{v}(t)$) and vice versa using the simple Kepler's laws with respect to the focus.

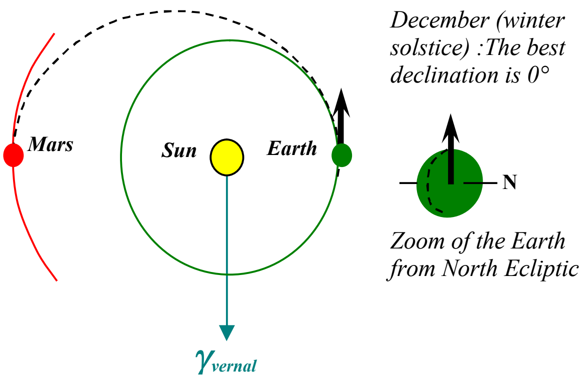
The hyperbolic branch of the hyperbola has a direction that is defined by the declination ($\delta°$) with respect to a reference plane and the right ascension ($\alpha°$) from a reference axis in the reference plane. The same parameters define the direction of the infinity velocity vector $\mathbf{V}_\infty$.



Note : For a travel from planet to planet, roughly, the lowest cost trajectory in terms of delta V constrains the declination with the date of the travel:

From Earth and for a date of the travel around summer or winter solstices (June or December) the declination is roughly zero. For a date around spring equinox (March) the declination is roughly -23°. For a date around autumn equinox (September) the declination is roughly +23°.





Integration

The differential equation of the second order is first transformed into two first order equations (1 vector equation for the velocity and the other for the acceleration): for the movement of a satellite around a focus, one have 7 equations to be solved simultaneously (one is trivial for the time) or 8 equations when one have to take into account the satellite mass.
With Runge-Kutta fifth order RK5, one integrate the second order differential equation system as a first order double size system, the subscript 1 being relative to the focus, the subscript 2 being relative to the satellite:

$$\frac{d\vec{\mathbf{r}}_{12}}{dt} = \vec{\mathbf{v}}_{12}$$

Transform second order differential system into one order double size system

$$\frac{d\vec{v}_{12}}{dt} = -G\frac{m_1}{r_{12}^3}\vec{r}_{12} + \sum \vec{\text{Thrust}}\Big/ m_2 + \sum \vec{\text{Perturb}}\Big/ m_2$$

Term ($m_2 + m_1$) rounded to m_1. Sum of vectors

$$\frac{dm_2}{dt} = -\frac{\sum\left\|\vec{\text{Thrust}}\right\|}{g_0.\text{Isp}}$$

Sum of the norms values

$$\frac{d\mathbf{t}}{dt} = \mathbf{1}$$

This equation is trivial, it simplify the Runge-Kutta code, and reduce the error risk.

Important note:

The perturbation terms in the equation for the derivative of the velocity (as described at the chapter beginning) are complete for any perturbing body. This allows to integrate the CR3BP (circular restricted 3 body problem) without any discrepancies. See the last annex.

Runge-Kutta process in few words:

*The numerical integration process of ODE (Ordinary Differential Equations) is simply coming from the fact that the initial value of the vector **u0** at time t0 is known and that the derivative vector **du**/dt is a known function (function of the vector **u** and the time). Thus the step further is given thorough: **u**(t0+dt) = **u0** + dt . **du**/dt and so on.*

This formulation is the Taylor development at the first order. In spite of the fact that the Taylor writing is very simple, the disadvantage of that simple form is the poor performance (the step time shall be enough small to get right results) and there are no real means to evaluate the process stability: the reason being that the process does not take into account all the possible information included in the known function.

*The Runge-Kutta process is based on the same principle, except that the known function **du**/dt is intensively used to compute successively parts of the increment of the vector **u** that are after used to compute other parts of the vector **u,…** before computing the final result **u**(t0+dt). The information included into the known function is thus fully available to provide an accurate result. Moreover one can know in advance what is the real "good" size of the step time (Fehlberg*[R 1]*).*

- *Integration step: this is a very clever feature of the program to be able to discriminate automatically how the step size must evolve with respect to the integration errors: errors are computed thanks to a comparison between integration **5th** order and integration 4th order. The process (Felhberg) is described in the reference **Numerical recipes***[R 1].
- *But it must be highlighted that the engineering signification of each integral equation is taken into account in the program for further processing, i.e, taking into account the individual units of each equation, and then that errors of same unit are square summed together. Engineering speaking, the velocity errors must be much lower than the distance errors because velocity is of course integrated to get the distance.*
- *Distances are summed together and velocities are summed together.*
- *A final criterion has been set up via the formulae: Epsilon=Max ($Eps_{distance}$, $Eps_{velocity}$ * 64 dt) which means that the errors in distance and velocity may become similar after 64 actual time steps.*

Eclipse

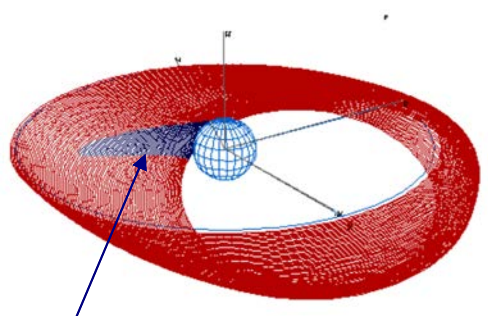
The process used to compute the Sun eclipse (and thus, the satellite enter in the Sun shadow) is described in the following points:

1 Computation of Earth position in the inertial heliocentric ecliptic frame ([R 7] Bouiges formulae) …
2 Computation, in inertial heliocentric ecliptic frame, of the position of the cone summit (along Sun -> Earth vector and along Sun+Rsun_ez --> Earth+Rearth_ez vector
3 Computation of the position of the cos (cone angle) (by the scalar product Sun -> earth vector and Sun+Rsun_ez --> earth+Rearth_ez vector)
4 Transform to inertial geocentric equatorial frame
5 Computation of vector cone--> Earth : RconeEarth
6 Computation of vector RconeSat = RconeEarth + R
7 if RconeSat ² > Rconeearth ² then SUN
8 if not: compute the satellite view angle from the cone : cos(angle)=RconeSat.RconeEarth/(|RconeSat||RconeEarth|)
9 If cos(angle) > cos (cone angle) then ECLIPSE

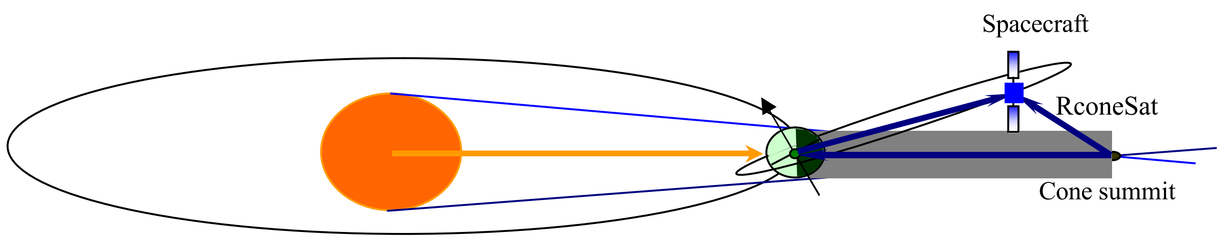
When the eclipse checkbox is checked (in the TriaX window called Manoeuvre), and if the eclipse status is true then if needed, the propulsion is turned off.
The corresponding event is written in the Warning log textbox.
If needed, the propulsion is automatically turned on again when the status eclipse becomes false, and the event is written with the duration of the non-propulsion.

Note: the duration of non-propulsion due to the eclipse as indicated at the eclipse off event is rough value at +-one Runge-Kutta step time.

Note: if the eclipse occurs twice per orbit during the thrust arc, *the summary results may indicate only the last eclipse duration of non-propulsion (for example if, when the thrust arc start at apogee, the eclipse occurs before and after apogee). In order to get meaningful results of the eclipse duration of non-propulsion, the user shall try to have only one eclipse occurrence per thrust arc (start the thrust at perigee in the example). But the timestamps of the events written in the log textbox are correct.*



Note: the non-propulsion during eclipse period is pointed out with a "**blue night**" colour in the 3D plot.



Example of the configuration Sun, Earth , shadow cone summit and Earth Spacecraft on an elliptical orbit.

Lagrangian points

The process used to compute the Lagrangian point L1 or L2 is based on the solution r_{mLi} of the following equations:

$$\frac{\mu_E}{(r_{Em}-r_{mL1})^2}-\frac{\mu_m}{r_{mL1}{}^2}=\frac{(\mu_E+\mu_m)\cdot(r_{Em}-r_{mL1})-\mu_m r_{Em}}{r_{Em}{}^3} \quad \text{and} \quad \frac{\mu_E}{(r_{Em}+r_{mL2})^2}+\frac{\mu_m}{r_{mL2}{}^2}=\frac{(\mu_E+\mu_m)\cdot(r_{Em}+r_{mL2})-\mu_m r_{Em}}{r_{Em}{}^3}$$

where r_{Em} is the distance Earth to Moon (subscripts respectively E and m). Simple Zbrent (r_{mLi}) routine is used.

History

The program presented here is based on a 26 years history.
The early beginning of such program came with the need to be able to represent in three dimensions a constellation like the GPS one. The basis of TriaX program was born. Contrary to the other three dimensional software, the method used here relied on the intensive use of scalar product and vector product (for all the projections of the space on a plane). By prohibiting the classical use of trigonometric routines, the computation time was largely improved. The computer language used was Quick basic.

The second step of the program came with the need to compute the Delta V losses due to long thrust arcs. This was achieved with a Runge-Kutta 4th order (RK4) integration method. In order to perform faster computation, only two dimensions were used at that time with the polar co-ordinates r, r_dot, θ, θ_dot. The validation of the program was based on the quantification of the integration process errors.

The third step came with the need to represent the North-South station keeping of a geostationary satellite. A three-dimension integration process was then based on the full Cartesian co-ordinates of r(t) and v(t).
In order to be able to represent clearly the changes during such manoeuvre, a set of visualisation 3D features were added. Cylindrical amplification factor (along Z axis) for small inclination views, Altitude zooming near the reference GEO orbit, Cubic views (with the three projections added simultaneously with the main drawing)… are some of those special features. Integration process was improved using RK 5 with automatic error / time step adjustment. RK 8 also included was not found to be very interesting.

The program was intensively used for performing GTO – GEO orbit transfer with high specific impulse and low thrust propulsion systems. Plots of that program were used in the description of Snecma Patents, and in many papers presented to international conferences.
Also, the landing of a spacecraft on the moon surface was computed accurately in order to be able to evaluate the so called "deltaV losses" (used in the course of the Euromoon Esa program).
Interplanetary manoeuvres (Rosetta Wirtanen) have been computed and presented in paper at international conferences.

The whole program was transformed into the Visual Basic environment for a user-friendly package. The routines dealing with the 3D visualisation were then built into an independent general purpose DLL, for further use in other software as a GlobalMultiUse DLL.

A special release of the program has been built for integration into Power point presentation for showing any process of trajectory computation dynamically.

The results files maintain compatibility with 3DD viewer tools (TriaXExcelPro for example).

The full process is compatible for adjustment of one parameter in order to fulfil a given goal. This is applied for adjusting an initial orbit going to exactly to Earth-Moon libration point L1 or L2.

Since 2006, the company Kopoos Consulting Ind. is the copyright owner of this software. A continuous process of improvement is performed along years.

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- *…*

Annex 2 : Results files.

For the traceability of the computations performed, a files system is produced and managed by the program. The content and the meaning of the files is precisely defined in this annex.

The programme produces two kinds of results files, stored in the user subdirectory "…**xls_doc**\":

- A **summary file** called "**Res_Tmp.xls**" in the subdirectory "**xls_doc**\" of the program directory. This file contains, in text form --i.e. ASCII--, all the orbital data and parameters settings each time the thrusters are activated (or at each full orbit when the continuous thrusting strategy is selected). *Note: the file is* *<u>renewed for each new run of the program</u>.*
  *However, for traceability purpose, a file so-called "**Results.doc**" in the same subdirectory keep a trace of all the summary information generated by the run performed.* ***This file may increase in size, so it is recommended to delete that file from time to time or to rename it.*** *One can open it directly with Excel or also within Microsoft® Word -- orient the page in landscape and choose a small font size-- to select the part to analyse and to paste into Excel (with Excel, use the space as delimiter during the conversion text to column process).*
- A **detailed text file** called "**TriaxPol.xls**" in the subdirectory "**xls_doc**\" of the program directory. This file contains all the results of the computation steps, at each time step. **This file is a temporary file and may be very large depending on the run performed**, that is now no more an issue for its storage but may slow down the computations: in the contextual window Interactive TriaX, a check box "write TriaxPol" checked by default allows, when unchecked, to suspend the writing on that file until it is checked again. .
  *Note: the file "**TriaxPol.xls**" is however renewed for each new run of the programme.*

*Note: A **detailed file** called "**TriaxCarT.dat**" in the subdirectory "**data**\" of the working directory. This file contains all the results of the computation steps, at each time step. This file may be very large, so the data are stored in **compressed** form. Note: the file "**TriaxCarT.dat**" is renewed for each new run of the programme. A tool is provided by the program at "Menu / Option / **Results in Text file**" in order to generate the text file "**TriaxPol.xls**" for compatibility with previous releases of the programme.*

Note: the text file with the extension "**.xls**" can be opened directly with Microsoft® Excel for further analysis with two dimensional plots (or also a three dimensional plot with an additional program, like TriaXExcelPro). It is recommended to use the space as delimiter during the conversion text to column process of Excel.

"Res_Tmp.xls" in …\ xls_doc \

This is a temporary events results file (for traceability reasons, the same data are also added to "Results.doc"). For each event in the manoeuvre, a new line of data is written.
Content of one line of events results:

1	2	3	4	5	6	7	8	9	10	11	12	13
ecc	Altper $_m$	Altapo $_m$	inclin°	Omega°	w°	phi°	O+w+p°	w+p°	period s	Time(s)	dt(s)	Mass kg

14	15	16	17	18	19	20	21	22	23	24	25	26
Thrust N	half-arc°	beta-i°	Moon°	AnoEcc°	n°Arc	O+w°	mid°	alpha-i°	Eclip(s)	sma (m)	dV (m/s)	Lo°

27	28	29	30	31	32	33	34	35				
ra_sma°	d_sma°	e_x	e_y	i_x°	i_y°	TimeJD	MeanLo°	HourDN°	Etc...			

The content of each line is almost explicit with the following abbreviation:

- O, Omega : the RAAN, also called longitude of the ascending node,
- w : lowercase omega, the perigee argument,
- p, phi : the true anomaly,
- w+p : composite angle sum of the angles omega + phi, sometimes called "argument of latitude",
- O+w : composite angle sum of the angles Omega + phi, sometimes called "longitude of perigee ϖ ",
- sma : semi-major-axis of the orbit at the time of the event (meters),
- dt : the time step of the Runge-Kutta Integration (dt is not a constant value, the last value is shown),
- Period : period of the current orbit at the time of the event, computed using the Kepler formulae without perturbing terms (in second). One shall mention that when non Keplerian effects are considered, the formula is not accurate. $$period = 2*\pi\sqrt{\frac{sma^3}{G*m_{focus}}}$$
- Important: *those data are transformed with simple Keple'sr laws wrt the focus from the radius and velocity wrt the focus coming from the integration of the trajectory. Hence those data do not include the effects of the perturbations neither the perturbing bodies effects. As consistency the initial data are also managed into radius and velocity wrt the focus using only the same simple Kepler's laws.*

The thrusters are fully described by the thrust and the integral mass flow exhausted (given by the mass evolution). The specific impulse (Isp) can be deduced from the above data. Moreover, the cumulative value of the ideal velocity increment provided by the propulsion, since the start of the computation, is displayed in the column “dV (m/s)”. The thrust arc strategy is described with the half thrust arc “half-arc°”, the location of the middle thrust arc “mid°” and the out-of-orbital plane orientation (beta-i°) and its in-orbital plane orientation (alpha-i°). For some Moon capture trajectory, it is interesting to keep trace of the angular Moon position with respect to the vernal axis. The eclipses (Sun shadow) are traced as well for what corresponding to the non-propulsion duration due to the eclipse.

- For some geostationary satellite studies, the longitude of the spacecraft with respect to the rotating Earth is displayed in the column “Lo°”.
- For other studies, it is interesting to know the coordinates of the apogee (with its right ascension and its declination).
- For Geosynchronous satellites, it is interesting to use the eccentricity vector e_x=ecc*cos(Om+w); e_y=ecc*sin(Om+w) and inclination vector i_x°=inclin°*cos(Om); i_y°=inclin°*sin(Om))

In addition the JulianDay number (TimeJD) is given for each events.
The mean Longitude (MeanLo°), which is the longitude trend without taking into account the eccentricity effects on the longitude, is finally displayed is the events file. Like for the true longitude, the mean longitude is meaningless when the focus is not the Earth.

For some cases, the Hour of the descending node of the orbit (HourDN) is given. This is a help when working on near Earth Sun-synchronous orbits.

Note: to perform a new computation from one of the events line, a feature is included in the software. One simply copy one line (from the first excel cell to the last one, or directly from the text) into the corresponding textbox of the TriaX window called Configuration, double-click on that textbox for enabling the new data, and in the TriaX window one shall enable the Advanced features checkbox, and then the feature Paste values enabling the new data of the orbit and the new JulianDay of the event.

“Results.doc” in ...\ xls_doc \

This is a traceability file containing all the individual results of “Res_Tmp.xls”. The format is identical to the previous one. From time to time, it is recommended to rename that file or to delete it because it may become large (containing all the events of all the cases computed).

“Warning_Tmp.xls ” in ...\ xls_doc \

The file “Warning_Tmp.xls” is a copy of all the messages displayed during the computation in the *log textbox*. **This file is a temporary file.** When eclipses occurs (and when the corresponding check box has been checked), it is sometime more accurate to use the results of this file for computing the total “eclipse duration (without propulsion)”. The format with the extension “xls” is suited for making all computations on Excel.

"TriaxPol.xls" in ...\ xls_doc \

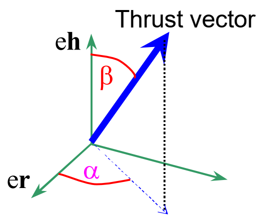
This file is a temporary file. The text file "**TriaxPol.xls**" in the subdirectory "**\xls_doc**" of the program directory is generated automatically. It contents, for each integration step, the following results available in text form (easily formatted with Excel, and fully compatible with the tool ***TriaXExcelPro***). There are 41 columns and more depending on the case ran. For keeping full trace of the settings, one can also open Res_Tmp.xls.

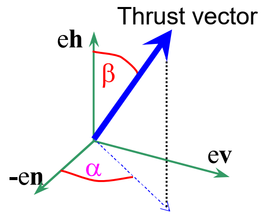
Time_d	Time_s	r_m	dr/dt_m/s	Th°	dTh/dt_°/s	Inclin°	Omega°	Mass_kg
Ecc	PerAlt_m	ApoAlt_m	omega°	phi°	Period_s	sma_m	C_m².rd/s/kg	Energy_J/kg
Vinfinite_m/s	r_x_m	r_y	r_z	v_x_m/s	v_y	v_z	Beta°	xi°
psi°	Alpha°	XIref°	PSIref°	XXref°	YYref°	ZZref°	Force_N	Isp_s
Lo	ExpleQuater_w	Quater_x	Quater_y	Quater_z	Etc...			

Date_Excel_d: Spacecraft Elapsed Time (days), starts from the date of the" windows: orbit or manoeuvre",
Time_s: Spacecraft Elapsed Time (s), the zero being set at the start when the programme run,
r_m : Radius from the focus to the spacecraft (m),
dr/dt_m/s : radial velocity (m/s),
Th° : angle from the node, also called argument of latitude (degree),
dTh/dt_°/s : angular velocity around the focus (degree /s),
Inclin° : inclination of the of the osculating orbit wrt the main plane (degree),
Omega° : RAAN, angle of the ascending node wrt the main X Galilean axis pointing toward the vernal point (°),
Mass_kg: The spacecraft mass (kg)
Ecc: eccentricity (-),
PerAlt_m : Altitude of the pericentre of the osculating orbit(m),
ApoAlt_m : Altitude of the apocentre of the osculating orbit(m),
omega° : perigee argument, angle of the perigee wrt the node axis (degree),
Phi° : angle of the current location wrt to the perigee (degree),
Period_s ; orbital period computed using the Kepler formulae without perturbing terms (s),
sma_m : semi-major axis (m),

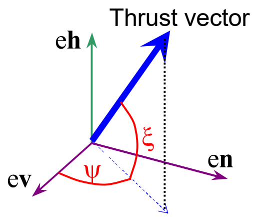
$$period = 2 * \pi \sqrt{\frac{sma^3}{G * m_{focus}}}$$

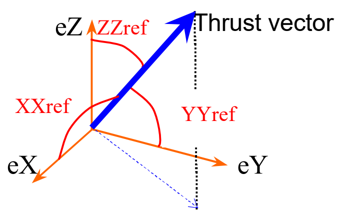
C_m².rd/s/kg : specific angular momentum constant (rd/s/kg),
Energy_J/kg : specific energy of the orbit (J/kg),
Vinfinite_m/s : infinite velocity in case of hyperbola conic (m/s),
Rx_km, Ry , Rz : Radius components from the focus to the spacecraft, in the inertial orientation frame (km),
Vx_m/s, Vy , Vz : Spacecraft Velocity components, in the inertial orientation frame attached on the focus (m/s),
Mass: Current weight of the spacecraft (kg),





Alpha°: angle of Orientation of the thrust wrt the Radius vector or the opposite of the normal to the velocity (°), right ascension,
Beta°: angle of the thrust wrt the Angular Momentum vector (°),
Xi°: complement of Beta to 90°, Xi° = 90° - Beta°. Angle of Orientation of the thrust wrt the orbital plane (°), declination,
psi°: angle of the thrust wrt the **Velocity** vector (°),
XIref°: angle of the thrust wrt the Z vector of the Galilean frame of Reference (°),
PSIref°: angle of the thrust wrt its orthogonal Plane, counted from the X vector of the Galilean frame of Reference (°),
XXref°: angle of the thrust wrt X vector of the Galilean frame of Reference with eX the direction of the vernal point (°),
YYref°: angle of the thrust wrt Y vector of the Galilean frame of Reference (°),
ZZref°: angle of the thrust wrt Z vector of the Reference --= 90°- XIref°-- (°),





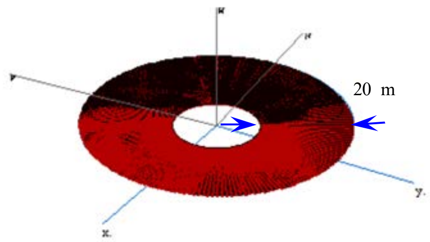
Force_N : Thrust force module (N),
Isp_s : specific impulse of the thruster(s),
Lo°: Longitude with respect to Greenwich Earth meridian (°),
ExpleQuater _w: One example of quaternion for the orienting the spacecraft Z axis with the orientation of the thrust (Quater_w=cos(ζ/2) and Quater_x=u_x . sin(ζ/2), Quater_y, Quater_z),
Etc...: depending on the case that is run, some other explicit results can be output .

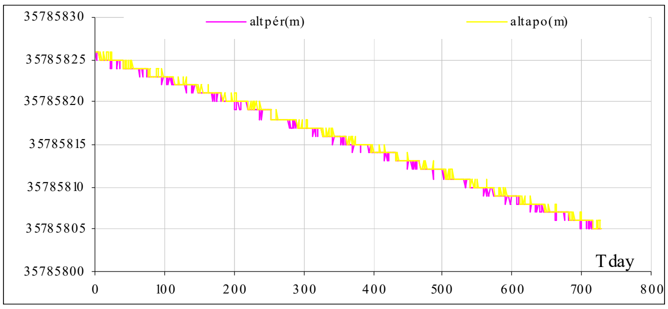
Annex 3: Integration accuracy

General purpose accuracy

Runge-Kutta Step 2° EpsilonRadius 0.5m EpsilonVelocity 0.0005m/s: After a 2 years integration (730 days) of a Geostationary Earth orbit without any perturbation, the accuracy is still better that 5 10^{-7} (i.e. a loss of 20 metres).

This kind of error is valid for the general purpose of the program.

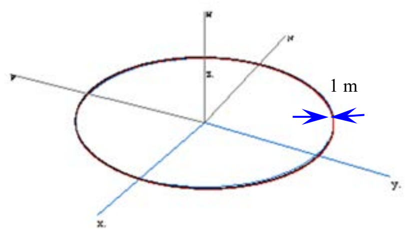


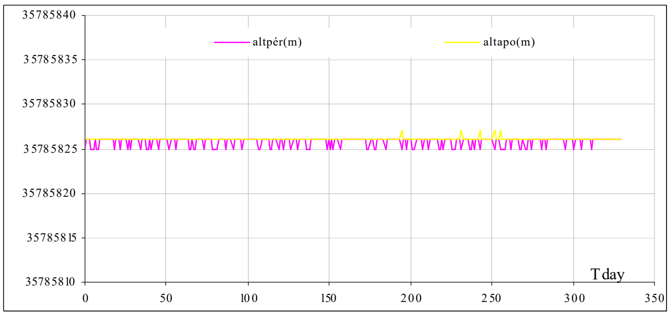


High accuracy

However, the precision can be improved by a fine tuning of the Runge-Kutta Step. More generally, the step can be optimised in order that the error is compliant with the given limits of Epsilon.

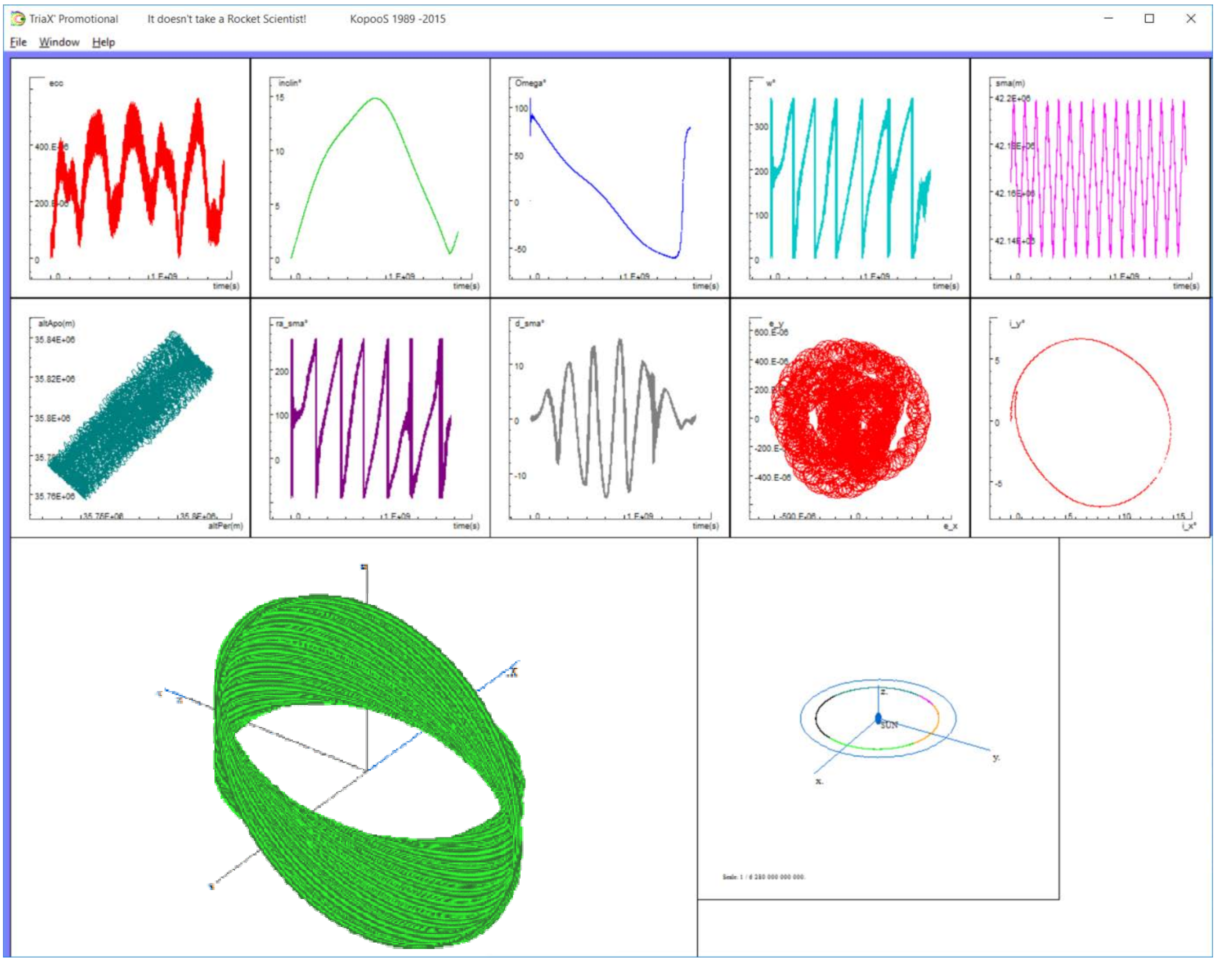
Runge-Kutta Step 0° (optimised) EpsilonRadius 0.0000005m (ie. 0.5µm): After a 1 year integration (330 days) of the same Geostationary Earth orbit without any perturbation and some minutes of computer time, the accuracy is better than 1 metre.

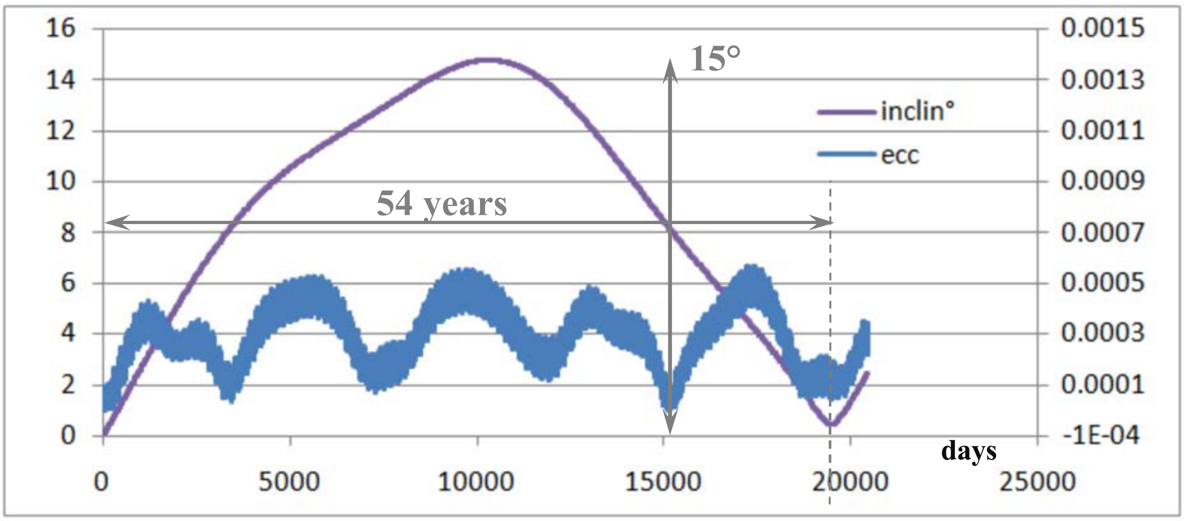




Other checks

A very well known behaviour of the Geosynchronous orbits is their evolution with an inclination that culminate at 15 ° in a cycle of 54 years (ref. [R 3] Agrawal, …). The computation of this case includes the **Moon**, **Sun** perturbations and the Earth oblateness (**J2**) perturbations. The simulation covering 21100 days (57 years) and one hour of computation shows very similar results.





Powerful checks within the Earth-Moon system CR3BP

TriaXOrbital is fully featured with the possibility to make the Moon orbit real (using Bouiges formulae [R 7]) or circular. This last case enable all simulations in the so called CR3BP (circular restricted 3 body problem)[R 20].

But contrary to almost any other programs dealing with Earth-Moon (E-M) libration points trajectories, TriaXOrbitaL is not using any special scheme in barycentric rotating coordinates for the integration of the dynamic equations, neither any normalization is needed, neither backward time integration (even if the tool is fully able to integrate backward in time): For Earth-Moon trajectories, TriaXOrbitaL is integrating forward time the dynamic equations with respect the main focus (Earth) in Centered Inertial frame (ECI) with of course the full perturbations from any other body selected. Hence it is important to show that the results provided by TriaXOrbitaL do not provide any deviation with respect to some known results.

First relevant check of Stability at L2

It is well known in the Earth-Moon system, that once the spacecraft is placed at L2 with the same velocity of L2 wrt Earth, it must stay there indefinitely. First a two impulses Transfer Trajectory is performed according to Farquhar [R 17] General Electric Co., "Lunar Libration Points Flight Dynamics Study Final rep.," NASA-CR-130135, Nov. 1968.

[R 18],[R 17]. Then a delta V at L2 is automatically added to the spacecraft for staying at L2.
But because L2 is unstable, any small perturbation in position or velocity will have large consequences for the further trajectory: the third body (the spacecraft) will be ejected soon from L2 in any cases.
The plots below are performed (automatically) in the ECI (Earth inertial frame) and in the synodic frame (rotating with respect to the Earth-Moon --those bodies are set along the x axis with y axis being in plane --)

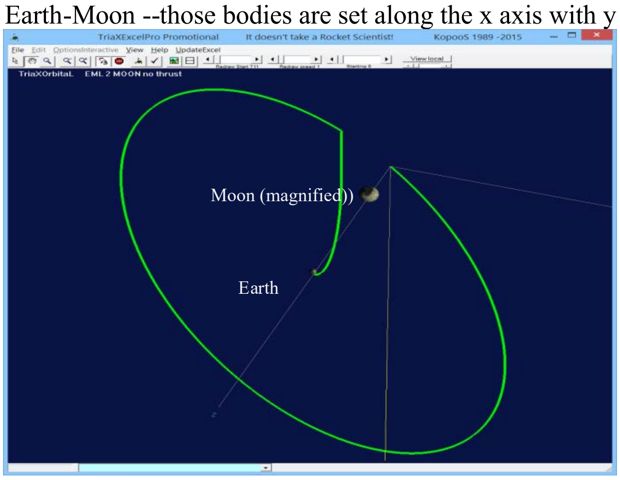


Figure 1: 3D view (under TriaXExcelPro) of the two impulses Transfer Trajectory followed by a stay at L2 for one Moon period, in ECI

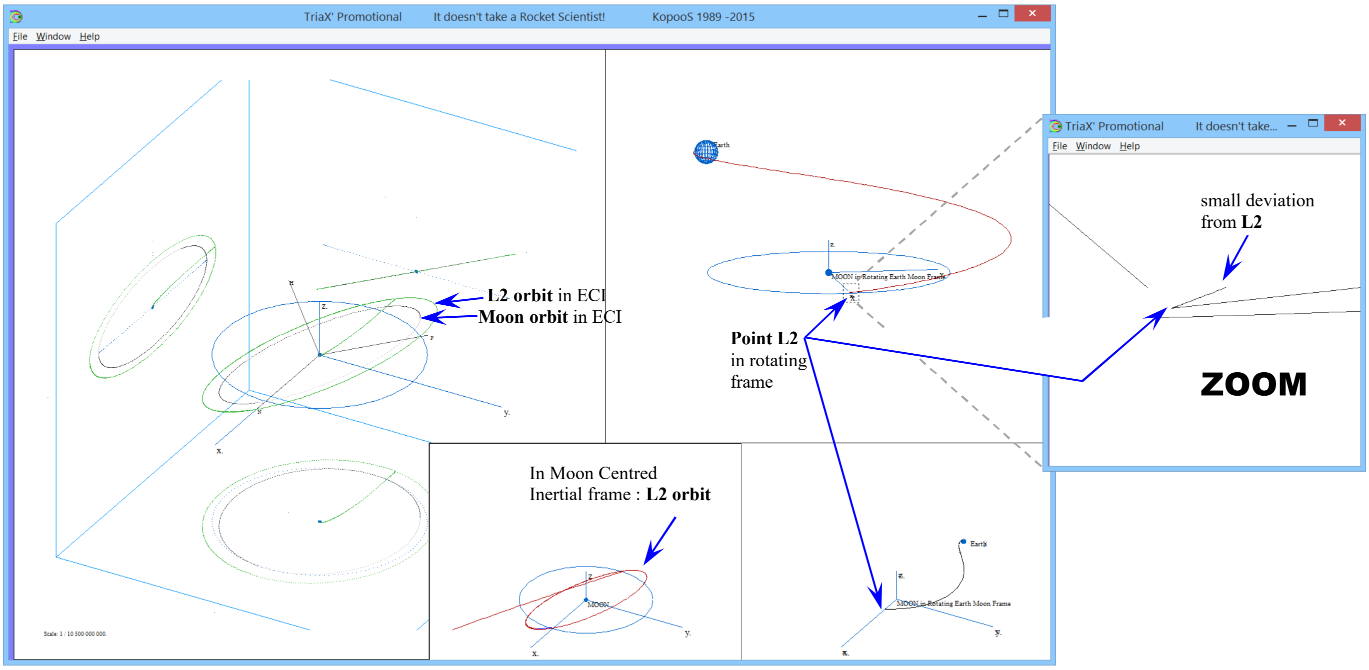


Figure 2: Orbits of L2 in ECI, in Moon Centered Inertial and Point L2 in the rotating synodic E-M frame

The duration at the point L2 before having any visible divergence in the trajectory of the spacecraft (due to integration errors) can be used for a measure of the fidelity of the integration.
For rough accuracy of Runge-Kutta (error of the order of 1 m per integration step), the stay is 30 days before shoving a discrepancy in the velocity of 1 m/s (that is the deltaV that would be needed for coming back to L2), see above the zoom of the rotating frame near the end of the x axis where the point L2 is located: a small deviation from L2 becomes visible.
The effect of much more accurate levels set for Runge-Kutta is plotted in the following graph : it appears that 30 days is already the sign of very high fidelity because for much more accurate integrations (up to integrations errors of 10µm at each step) , the stay at L2 culminates at 35.5 days.
This concludes the check successfully.

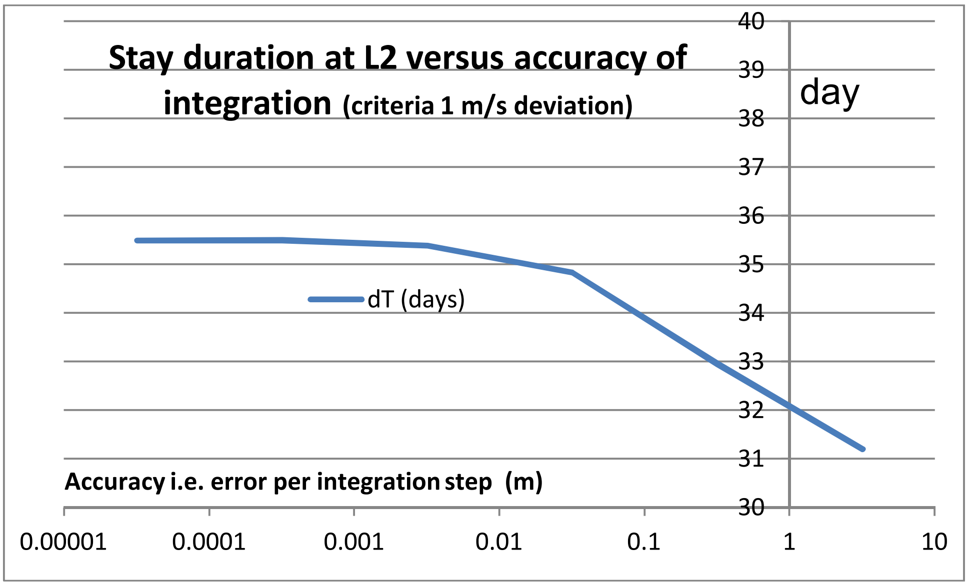
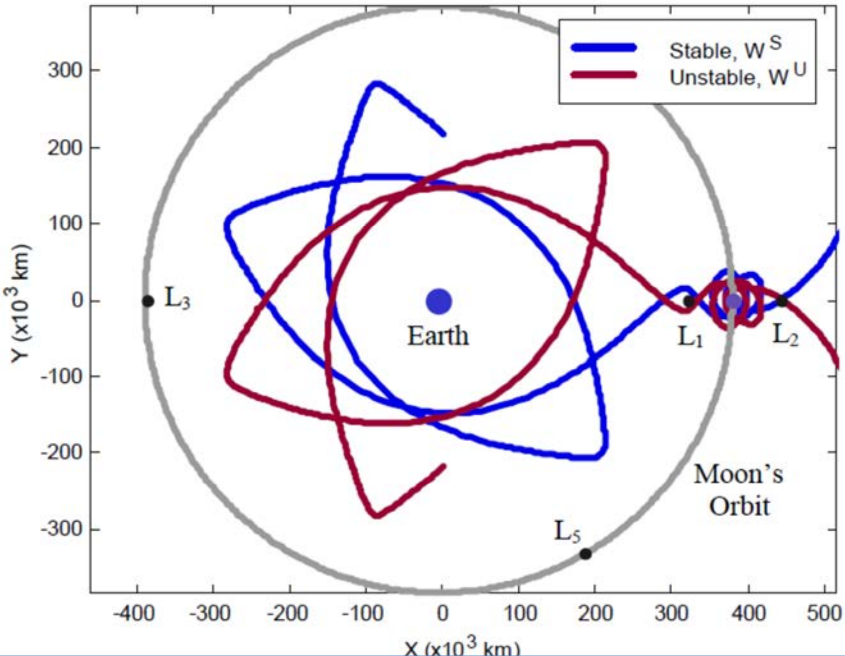


Figure 3: Evolution of the duration at L2 before deviation occurs versus integration errors levels

Second relevant check: Unstable manifold from L2

It is interesting to see in the references (for example [R 19] Thesis from Parker) what happen after a perturbation is intentionally provided to the third body (the spacecraft) that was staying at L2. In the following plot from Parker the perturbation lead the spacecraft to be ejected from L2 on the divergent trajectory so called the "unstable manifold (in red color) " in the rotating synodic E-M frame.



Parker-

Figure 4: Parker thesis plot, ref [R 19]: Jeffrey S. Parker thesis "Developing a Mission Design Architecture for the Earth-Moon Three-Body System," December 3, 2004

After the previous check of TriaXOrbital, the natural perturbations due to integration errors provides indeed systematically the same trajectory on the so called unstable manifold.

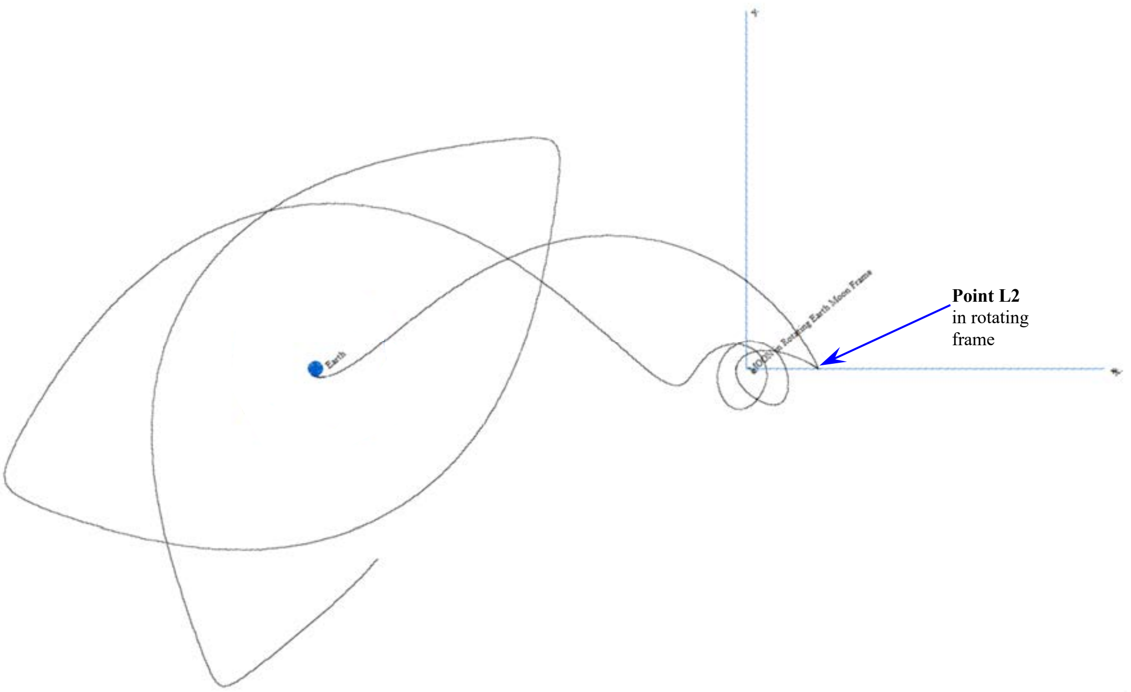


Figure 5: TriaXOrbital plot (starting from a two impulses Transfer Trajectory followed by a stay at L2 for 30 days) and diverging L2 on the unstable quasi-invariant manifold

Moreover, a continuation of the integration of the equations allow to further continue the trajectory along a converging manifold i.e. stable invariant manifold (in blue in the Parker's plot above, where it was needed for Parker to integrate backward in time from L2) that ends with TriaXOrbital near L2 (on the stable invariant manifold the point L2 is reached "exactly").

Note that with TriaXOrbital no switch is needed for selecting the red or blue invariant manifold (specialists of the CR3BP would say that like an homoclinic connection appears automatically with the tool), but the L2 point is not reached exactly, actually very near, see the following plot zoom.
This concludes the checks successfully.

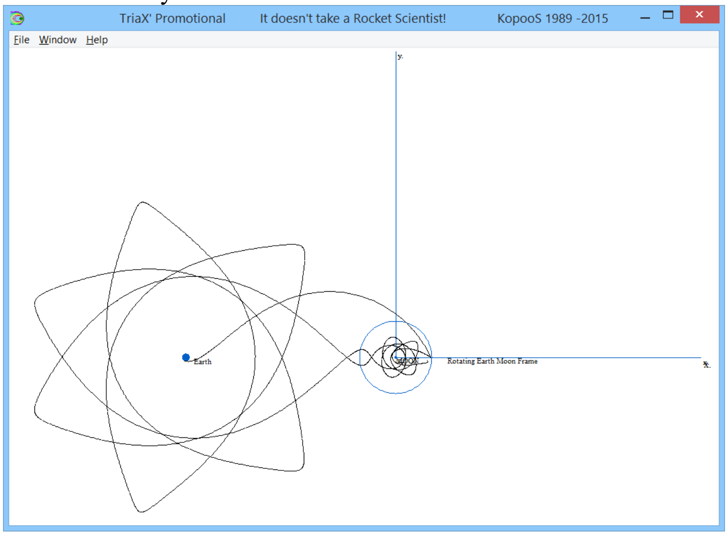


Figure 6: TriaXOrbital plot (from a two impulses Transfer followed by a stay at L2 for 30 days) diverging from and converging again back to near L2

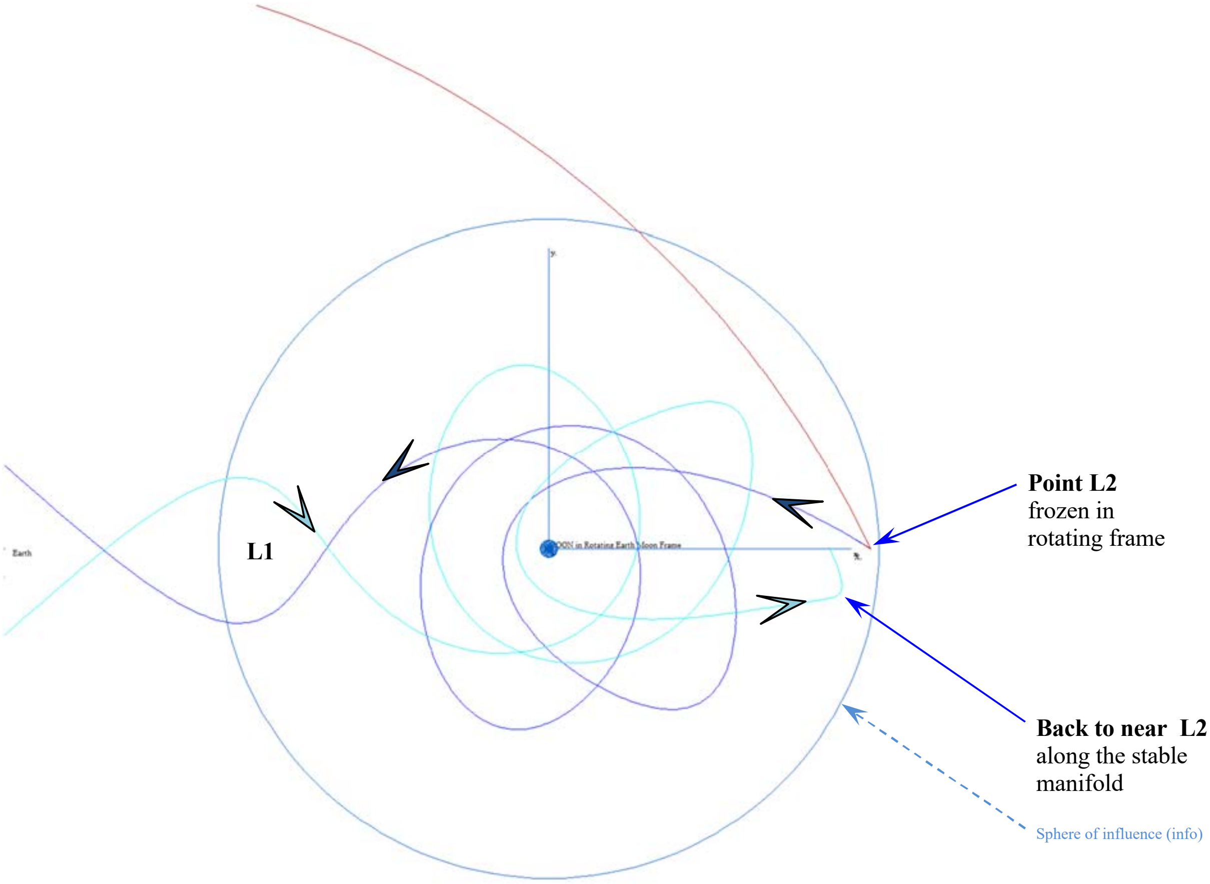


Figure 7: ZOOM of TriaXOrbital plot at L2 in the rotating frame: diverging L2 and converging back again near L2

Final note: It is to be highlighted that the rotating E-M synodic frame exhibits some specific looping and star shapes that are difficult to understand without a conventional plot of the third body (the spacecraft) into ECI. This is performed in the next plot: clearly, the star with 6 orbits followed by the spacecraft after L1 neck and before going back again to near L2 through a proximity with L1 neck are quite "normal ellipses" with respect to Earth.

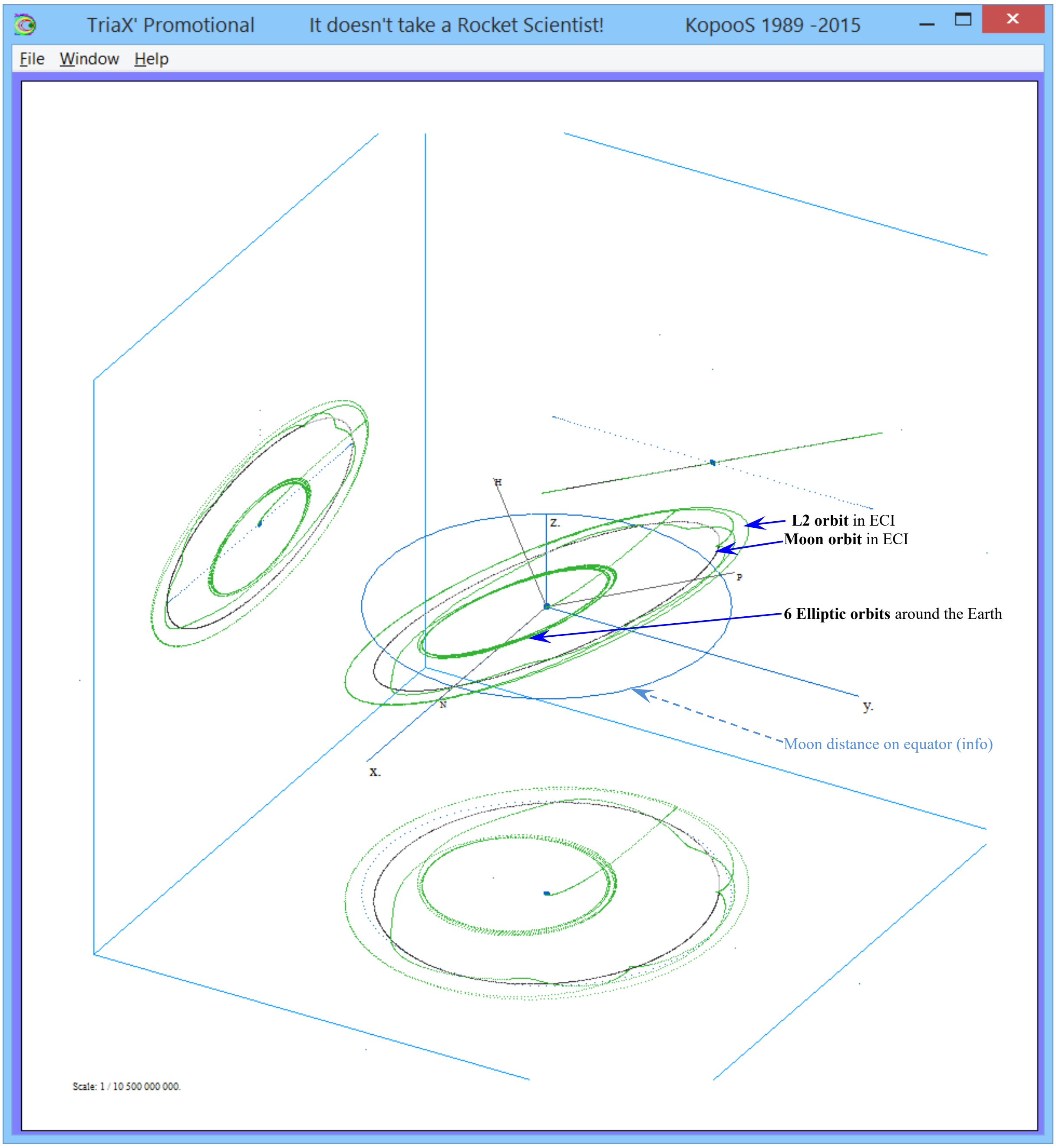


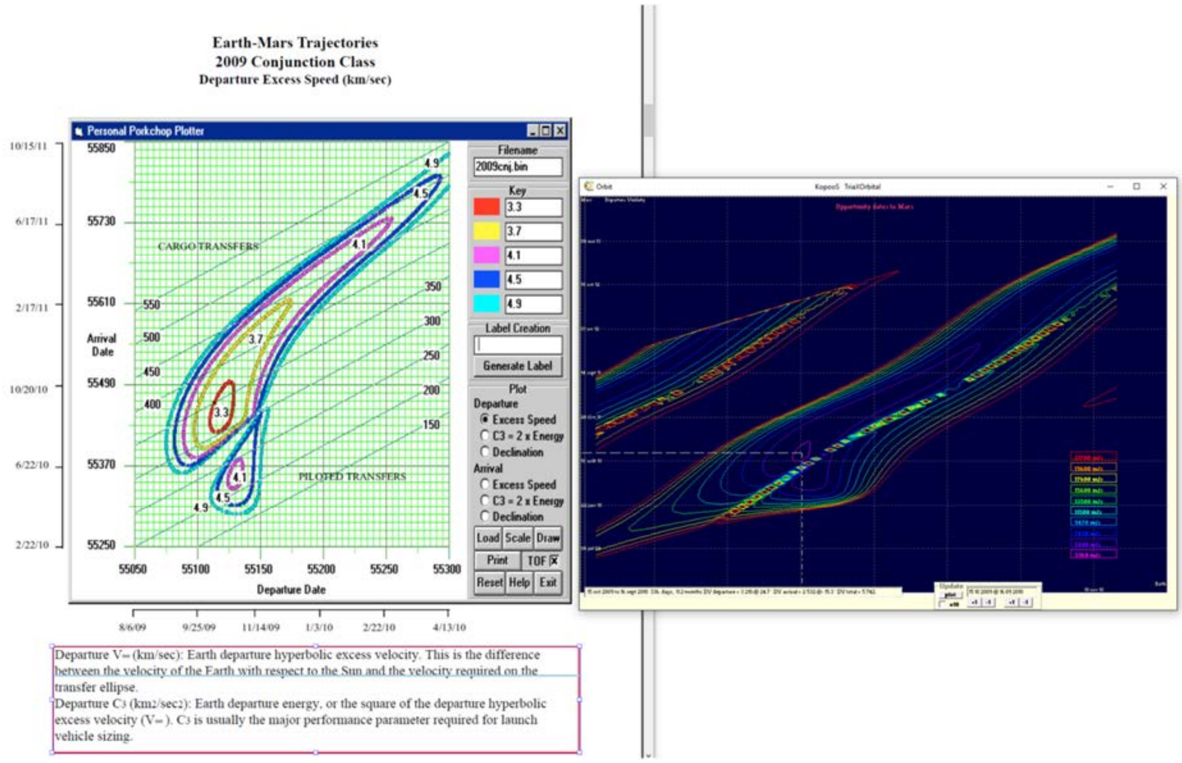
Figure 8: TriaXOrbital plot (from a two impulses Transfer followed by a stay at L2 for 30 days) diverging L2 in ECI

The 6 "elliptic orbits" around the Earth have the same inclination as the Moon, and it is to be highlighted that they have very similar orbital parameters with roughly perigee altitude of 125 000 km and apogee altitude of 290 000 km. Hence, those orbits are very well suited for a departure to the libration point L2 without any propellant cost (except very slight corrections) and thus specially no high impulses are needed at injection Moon or L2.
This is particularly interesting for Earth to Moon transfers when using low thrust electric propulsion.

Porkchop checks

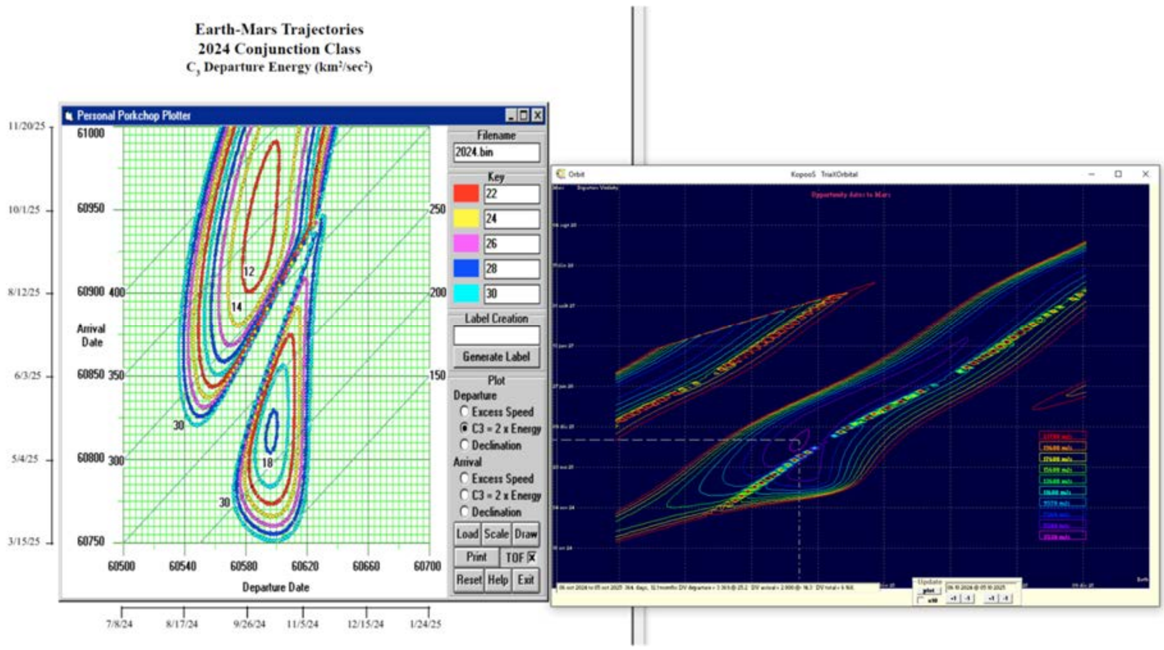
Check of Porkchop of TriaXOrbitaL wrt reference.
Ref. NASA TM—1998–208533 Interplanetary Mission Design Handbook ... 2009-2024; July 1998

- Case 2009



TriaXOrbitaL: 15 oct 2009 to 16 sept 2010 336. days, 11.2 months DV departure = 3 210.@ 24.7 ° DV arrival = 2 532.@- 15.3 ° DV total = 5 742 , **==> DV departure = 3.21 km/s while NASA mention <3.3 km/s.**

- Other case 2024



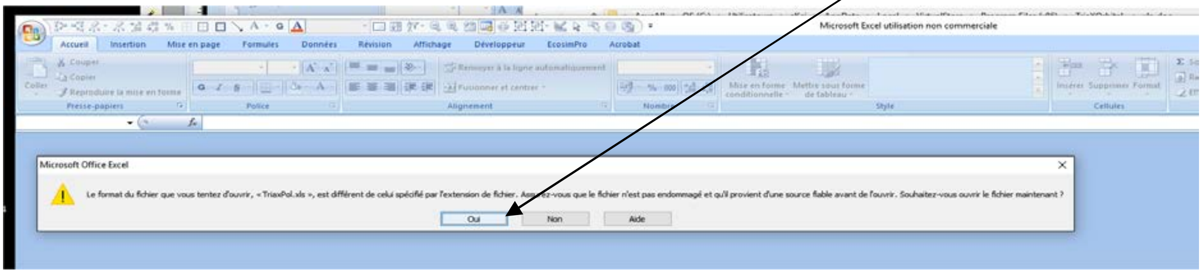
TriaXOrbitaL: 06 oct 2024 to 05 oct 2025 364. days, 12.1 months DV departure = 3 369.@ 25.2 ° DV arrival = 2 800.@- 14.3 ° DV total = 6 168. **C_3 departure= 3.369^2=11.35 km²/s² while NASA mention <12.**

- **Conclusion the opportunities are the same**

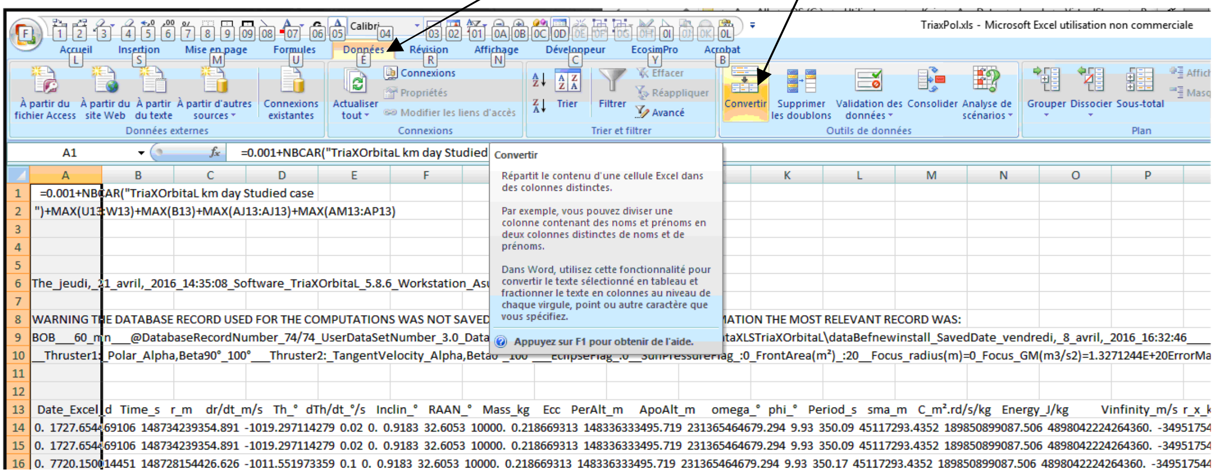
Annex 4: Excel files management

Opening "TriaXPol.xls" into Excel

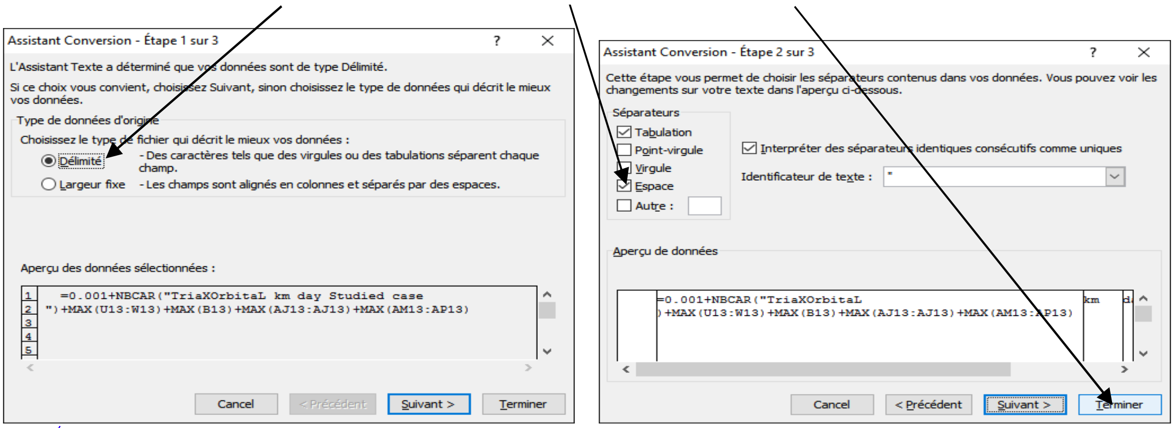
1. Double click on the file "TriaXPol.xls" to open into Excel
2. Message Excel: the file is a text file with extension xls, so Select YES to Open



3. Select the entire column A; Use DATA folder and select Conversion



4. Select "DELIMITER" and then select SPACE and Click on END



Final result: all data are in Excel columns